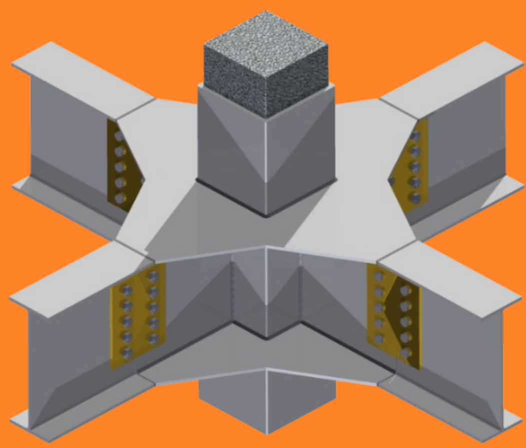
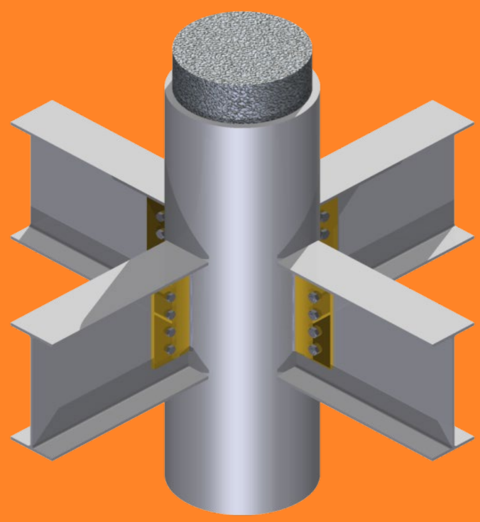
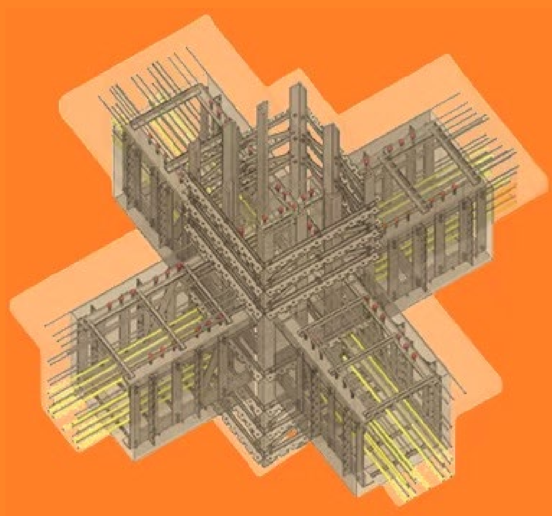
EXAMPLE CALCULATIONS – to the Requirements of BC4: 2025







Building and Construction Authority

Example Calculations to the Requirements of BC4: 2025

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1 Introduction

This guidebook presents five worked examples that provide a step-by-step procedure for determining the design resistance of Concrete-Filled Steel Tubular (CFST), Concrete-Encased Steel (CES), and Prefabricated Steel-Reinforced Concrete (PSRC) columns, in accordance with the requirements of BC4: 2025 – Design Guide for Steel-Concrete Composite Columns with High-Strength Materials.

- Example 1 demonstrates the calculation of the axial buckling resistance of a circular CFST column under pure compression.
- Example 2 evaluates a circular CFST column with an encased UC section and reinforcement, subjected to combined compression and uniaxial bending.
- Example 3 covers the design of a rectangular CFST column under combined compression and biaxial bending.
- Example 4 examines a square CES column with an encased UC section and reinforcement, under combined compression and uniaxial bending.
- Example 5 illustrates the design of a Prefabricated Steel-Reinforced Concrete (PSRC) column.

For Examples 1 to 4, different combinations of steel and concrete strengths are explored to evaluate the benefits of using high-strength concrete (HSC) and high-tensile steel (HTS). For PSRC columns (Example 5), the use of materials is limited to concrete grades up to C50 and steel grades up to S460, in line with current design provisions.

1 Example 1

1.1 General

In Example 1, the axial buckling resistance of a concrete filled steel tubular (CFST) column subject to pure compression is determined. The dimensions of the CFST column are shown in Figure 1.

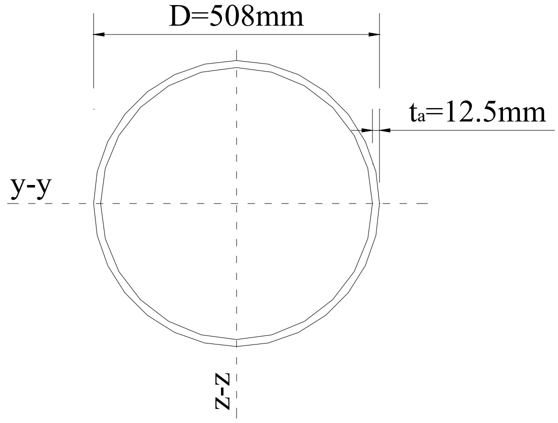


Figure 1: Cross-sectional dimensions of CFST column in Example 1

The column lengths and design loads are given as:

Column system length	L=4000 mm
Effective length	L_{eff}=4000 mm
Total design axial load	N_{Ed}=11000 kN
Design axial load that is permanent	$N_{G,Ed}$=4500 kN

To evaluate and compare their resistance, the following steel and concrete material grades are used:

a) CHS 508x12.5 - S355 steel + C40/50 concrete
b) CHS 508x12.5 - S355 steel + C90/105 concrete
c) CHS 508x12.5 - S460 steel + C40/50 concrete
d) CHS 406x12 - S460 steel + C90/105 concrete

1.2 CHS 508x12.5 - S355 steel tube infilled with C40/50 concrete

- Material

Concrete	C40/50, f_{ck}=40 N/mm^2

Steel tube Grade S355, f_y=355 N/mm^2

- Design strengths and modulus of material

Refer to Table 2.1 and Table 2.3 of BC4 for the characteristic strength of concrete and steel, and Table 1.1 of BC4 for the partial factors, the design strengths are determined as:

$$f_{yd} = f_y / \gamma_a = 355/1.0 = 355 \text{ N/mm}^2$$

$$E_a = 210 \text{ GPa}$$

$$f_{cd} = f_{ck} / \gamma_c = 40/1.5 = 26.7 \text{ N/mm}^2$$

$$f_{cm} = f_{ck} + 8 = 40 + 8 = 48 \text{ N/mm}^2$$

$$E_{cm} = 22\left(f_{cm}/10\right)^{0.3} = 22\left(48/10\right)^{0.3} = 35.2 \text{ GPa}$$

- Cross sectional areas

$$A_a = (\pi/4)\left[D^2 - (D-2t)^2\right] = (\pi/4)\left[508^2 - (508 - 2\times12.5)^2\right] = 19458 \text{ mm}^2$$

$$A_c = (\pi/4)(D-2t)^2 = (\pi/4)(508 - 2\times12.5)^2 = 183225 \text{ mm}^2$$

Unless otherwise stated, the subscript "a" stands for the steel section, and the "c" stands for the concrete section.

- Second moment of areas

$$I_a = (\pi/64)\left[D^4 - (D-2t)^4\right] = (\pi/64)\left[508^4 - (508 - 2\times12.5)^4\right]\times10^{-4} = 59755 \text{ cm}^4$$

$$I_c = (\pi/64)(D-2t)^4 = (\pi/64)(508 - 2\times12.5)^4 \times 10^{-4} = 267152 \text{ cm}^4$$

- Check for local buckling (refer to Table 3.1 of BC4)

$$D/t_a = 508/12.5 = 40.6 < 90\left(235/f_y\right) = 90\left(235/355\right) = 59.6$$

Resistance against local buckling is adequate!

- Long-term effect

The long-term effect could be evaluated in accordance with EN 1992-1-1: 2004. Herein, the simplified method given in guidebook by Liew and Xiong (2015) "Design Guide for Concrete Filled Tubular Members with High Strength Materials – An Extension of Eurocode 4 Method to C90/105 Concrete and S550 Steel" is referred to.

The age of concrete at the moment considered *t* is conservatively taken as infinity. For the age of concrete on first loading by effects of creep, although EN 1994-1-1 (2004) recommends t_0 = 1 day, it is actually the judgement of designer to determine t_0 since it makes quite a difference whether this age is assumed to be 1 day or 1 month. Herein, t_0 is assumed as 14 days, as said, it could be different. The relative humidity *RH* for infilled concrete is taken as 50%.

Perimeter of concrete section: $u = \pi(D - 2t_a) = \pi(508\text{-}2\times12.5) = 1517$ mm

Notional size of concrete section: $h_0 = 2\mathrm{A_c}/u = 2\times183225/1517.4 = 241.5$ mm

Coefficient: $\alpha_1 = (35/f_{cm})^{0.7} = (35/48)^{0.7} = 0.80$

Coefficient: $\alpha_2 = (35/f_{cm})^{0.2} = (35/48)^{0.2} = 0.94$

Coefficient: $\alpha_3 = (35/f_{cm})^{0.5} = (35/48)^{0.5} = 0.85$

Factor: $\varphi_{\mathrm{RH}} = \left(1 + \dfrac{1 - RH/100}{0.1\sqrt[3]{h_0}}\alpha_1\right)\alpha_2 = \left(1 + \dfrac{1 - 50/100}{0.1\sqrt[3]{241.5}}\times0.80\right)\times0.94 = 1.54$

Factor: $\beta(f_{cm}) = 16.8/\sqrt{f_{cm}} = 16.8/\sqrt{48} = 2.42$

Factor: $\beta(t_0) = 1/(0.1 + t_0^{0.2}) = 1/(0.1 + 14^{0.2}) = 0.56$

Factor: $\varphi_0 = \varphi_{\mathrm{RH}}\beta(f_{cm})\beta(t_0) = 1.54\times2.42\times0.56 = 2.08$

Factor: $\beta_H = 1.5\left[1 + (0.012RH)^{18}\right]h_0 + 250\alpha_3$

$= 1.5\left[1 + (0.012\times50)^{18}\right]\times241.5 + 250\times0.85 = 576$

Factor: $\beta_c(t, t_0) = \left(\dfrac{t - t_0}{\beta_H + t - t_0}\right)^{0.3} = \left(\dfrac{\infty - 14}{576 + \infty - 14}\right)^{0.3} = 1.0$

Creep coefficient: $\varphi_t = \varphi_0\beta_c(t, t_0) = 2.08\times1.0 = 2.08$

- Elastic modulus of concrete considering long-term effect (refer to Eq.(3.14) of BC4)

Concrete is sensitive to long-term deformations due to creep and shrinkage. To allow for this, the flexural stiffness of concrete section is reduced.

$$E_{c,eff} = \frac{E_{cm}}{1 + (N_{G,Ed}/N_{Ed})\varphi_t} = \frac{35.2}{1 + (4500/11000)\times2.08} = 19 \text{ GPa}$$

- Effective flexural stiffness of cross-section

$$(EI)_{eff} = E_a I_a + 0.6E_{c,eff} I_c = \left(210\times10^3\times59755\times10^4 + 0.6\times19\times10^3\times267152\times10^4\right)\times10^{-3}$$
$$= 1.56\times10^{11}\ \text{kN}\cdot\text{mm}^2$$

- Elastic critical Euler buckling resistance

$$N_{cr} = \frac{\pi^2 (EI)_{eff}}{L_{eff}^2} = \frac{\pi^2\times1.56\times10^{11}}{4000^2} = 96209\ \text{kN}$$

- Characteristic plastic resistance of cross-section

$$N_{pl,Rk} = A_a f_y + A_c f_{ck} = (19458\times355 + 183225\times40)\times10^{-3} = 14250\ \text{kN}$$

- Relative slenderness ratio

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} = \sqrt{\frac{14250}{96209}} = 0.385 < 0.5$$

- Confinement coefficients with load eccentricity *e*=0

Since the slenderness $\bar{\lambda}$ is less than 0.5 and the load eccentricity is equal to 0 (axially loaded column), the confinement effect may be considered for the circular CFST column.

$$\eta_a = \eta_{a0} = \min\left[0.25\left(3+2\bar{\lambda}\right), 1.0\right] = 0.942$$

$$\eta_c = \eta_{c0} = \max\left[4.9 - 18.5\bar{\lambda} + 17\bar{\lambda}^2, 0\right] = 0.298$$

- Design plastic resistance of cross-section considering the confinement effect (refer to Eq.(3.2) of BC4)

$$N_{pl,Rd} = \eta_a A_a f_{yd} + A_c f_{cd}\left(1 + \eta_c \frac{t}{D}\frac{f_y}{f_{ck}}\right)$$
$$= \left[0.942\times19458\times355 + 183225\times26.7\left(1 + 0.298\frac{12.5}{508}\frac{355}{40}\right)\right]\times10^{-3}$$
$$= 11727\ \text{kN}$$

It is worthwhile to note that the yield strength of steel is reduced ($\eta_a < 0$) and strength of concrete increases ($1+\eta_c \frac{t}{D}\frac{f_y}{f_{ck}} > 0$) with the consideration of confinement effect.

- Steel contribution ratio

$$\delta = A_a f_{yd} / N_{pl,Rd} = \left(19458\times10^{-3}\times355\right)/11727 = 0.59 < 0.9$$

- Imperfection factor

Refer to Table 3.3 of BC4, the buckling curve is taken as "a". Thus the imperfection factor α is determined as 0.21 according to Table 3.2 of BC4.

- Buckling reduction factor

$$\Phi = 0.5\left[1+\alpha\left(\bar{\lambda}-0.2\right)+\bar{\lambda}^2\right] = 0.5\left[1+0.21\times\left(0.385-0.2\right)+0.385^2\right] = 0.593$$

$$\chi = \frac{1}{\Phi+\sqrt{\Phi^2-\bar{\lambda}^2}} = \frac{1}{0.593+\sqrt{0.593^2-0.385^2}} = 0.957$$

- Buckling resistance

According to Section 3.3.1 of BC4, the axial buckling resistance is checked as:

$$N_{b,\mathrm{Rd}} = \chi N_{\mathrm{pl,Rd}} = 0.957\times11727 = 11223\ \mathrm{kN} > N_{\mathrm{Ed}} = 10000\ \mathrm{kN}$$

Buckling resistance is adequate!

1.3 CHS 508x12.5 - S355 steel tube infilled with C90/105 concrete

In this section, the normal strength concrete (NSC) C40/50 is replaced by high strength concrete (HSC) C90/105. The steel grade is not changed.

- Design strength

Refer to Table 2.2, Eq.(2.2) and Eq.(2.3) of BC4, the effective compressive strength and modulus of elasticity of the HSC are calculated as:

$$f_{ck} = 72\ \mathrm{N/mm^2}$$

$$E_{cm} = 41.1\ \mathrm{GPa}$$

$$f_{cd} = f_{ck}/\gamma_c = 72/1.5 = 48 \text{ N/mm}^2$$

$$f_{cm} = f_{ck} + 8 = 72 + 8 = 80 \text{ N/mm}^2$$

- Creep coefficient could be similarly determined as $\varphi_t = 1.29$.
- Elastic modulus of concrete considering long-term effect

$$E_{c,eff} = E_{cm}\frac{1}{1+\left(N_{G,Ed}/N_{Ed}\right)\varphi_t} = \frac{41.1}{1+\left(4500/11000\right)\times 1.29} = 26.9 \text{ GPa}$$

- Effective flexural stiffness of cross-section

$$\begin{aligned}(EI)_{eff} &= E_a I_a + 0.6E_{c,eff} I_c \\ &= \left(210\times10^3\times59755\times10^4 + 0.6\times26.9\times10^3\times267152\times10^4\right)\times10^{-3} \\ &= 1.69\times10^{11} \text{ kN}\cdot\text{mm}^2\end{aligned}$$

- Confinement effect and design plastic resistance of cross-section

$$N_{cr} = \frac{\pi^2 (EI)_{eff}}{L_{eff}^2} = \frac{\pi^2\times1.69\times10^{11}}{4000^2} = 104010 \text{ kN}$$

$$N_{pl,Rk} = A_a f_y + A_c f_{ck} = \left(19458\times355 + 183225\times72\right)\times10^{-3} = 20112 \text{ kN}$$

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} = \sqrt{\frac{20112}{104010}} = 0.44 < 0.5$$

$$\eta_a = \eta_{a0} = \min\left[0.25\left(3+2\bar{\lambda}\right),1.0\right] = 0.970$$

$$\eta_c = \eta_{c0} = \max\left[4.9-18.5\bar{\lambda}+17\bar{\lambda}^2,0\right] = 0.052$$

$$\begin{aligned}N_{pl,Rd} &= \eta_a A_a f_{yd} + A_c f_{cd}\left(1+\eta_c\frac{t}{D}\frac{f_y}{f_{ck}}\right) \\ &= \left[0.970\times19458\times355 + 183225\times48\left(1+0.052\frac{12.5}{508}\frac{355}{72}\right)\right]\times10^{-3} = 15562 \text{ kN}\end{aligned}$$

- Steel contribution ratio

$$\delta = A_a f_{yd}/N_{pl,Rd} = \left(19458\times10^{-3}\times355\right)/15562 = 0.444 < 0.9$$

- Buckling resistance

Buckling curve="a", α=0.21

$$\Phi = 0.5\left[1+\alpha\left(\bar{\lambda}-0.2\right)+\bar{\lambda}^2\right] = 0.5\left[1+0.21\times\left(0.44-0.2\right)+0.44^2\right] = 0.622$$

$$\chi = \frac{1}{\Phi+\sqrt{\Phi^2-\bar{\lambda}^2}} = \frac{1}{0.622+\sqrt{0.622^2-0.44^2}} = 0.942$$

Thus, the axial buckling resistance is:

$$N_{b,\mathrm{Rd}} = \chi N_{\mathrm{pl,Rd}} = 0.942\times15562 = 14659\ \mathrm{kN}$$

Compared with that using NSC C40/50, the buckling resistance is increased by:

$$\frac{14659-11223}{11223}\times100\%=30.6\%$$

By replacing C40/50 normal strength concrete with C90/105 high strength concrete, the axial buckling resistance of the CFST column is improved by 31%.

1.4 CHS 508x12.5 - S460 steel tube infilled with C40/50 concrete

In this section, the mild steel S355 is replaced by S460, the concrete grade is not changed.

- Confinement effect and design plastic resistance of cross-section

$$N_{pl,Rk} = A_a f_y + A_c f_{ck} = \left(19458\times460+183225\times40\right)\times10^{-3} = 16297\ \mathrm{kN}$$

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} = \sqrt{\frac{16297}{96209}} = 0.412 < 0.5$$

$$\eta_a = \eta_{a0} = \min\left[0.25\left(3+2\bar{\lambda}\right),1.0\right] = 0.956$$

$$\eta_c = \eta_{c0} = \max\left[4.9-18.5\bar{\lambda}+17\bar{\lambda}^2,0\right] = 0.166$$

$$N_{pl,Rd} = \eta_a A_a f_{yd} + A_c f_{cd}\left(1+\eta_c\frac{t}{D}\frac{f_y}{f_{ck}}\right)$$

$$=\left[0.956\times19458\times355+183225\times26.7\left(1+0.166\frac{12.5}{508}\frac{355}{40}\right)\right]\times10^{-3}$$

$$=13687\ \mathrm{kN}$$

- Steel contribution ratio

$$\delta = A_a f_{yd} / N_{pl,Rd} = \left(19458 \times 10^{-3} \times 460\right) / 13687 = 0.655$$

- Buckling resistance

$$\text{Buckling curve="a"}, \ \alpha = 0.21$$

$$\Phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right] = 0.5\left[1 + 0.21 \times \left(0.412 - 0.2\right) + 0.412^2\right] = 0.607$$

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} = \frac{1}{0.607 + \sqrt{0.607^2 - 0.412^2}} = 0.95$$

$$N_{b,\mathrm{Rd}} = \chi N_{\mathrm{pl,Rd}} = 0.95 \times 13687 = 13003 \text{ kN}$$

Compared with that with the mild steel S355, the axial buckling resistance by using S460 is increased by:

$$\frac{13003 - 11223}{11223} \times 100\% = 15.9\%$$

By use of steel S460 replacing S355, the axial buckling resistance of the CFST column is improved by 15.9%.

1.5 CHS 406x12 - S460 steel tube infilled C90/105 concrete

In this section, the normal strength materials are replaced by higher strength steel S460 and concrete C90/105. The compatibility between steel grade and concrete class should be evaluated in accordance with Table 2.7 of BC4. The purpose of using higher strength materials is to reduce the cross-sectional size of the original CFST column but the axial buckling resistance remains the same.

- CHS 406x12 with S460 steel is tried

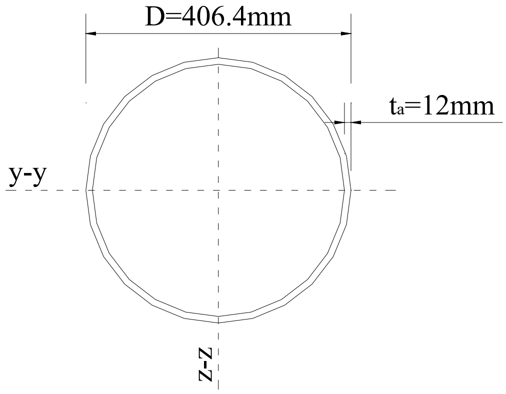


Figure 2: Reduced cross-sectional size

- Section properties

$$A_a = (\pi/4)\left[D^2 - (D-2t)^2\right] = (\pi/4)\left[406.4^2 - (406.4 - 2\times 12)^2\right] = 14900 \text{ mm}^2$$

$$A_c = (\pi/4)(D-2t)^2 = (\pi/4)(406.4 - 2\times 12)^2 = 114800 \text{ mm}^2$$

$$I_a = (\pi/64)\left[D^4 - (D-2t)^4\right] = (\pi/64)\left[406.4^4 - (406.4 - 2\times 12)^4\right]\times 10^{-4} = 28940 \text{ cm}^4$$

$$I_c = (\pi/64)(D-2t)^4 = (\pi/64)(406.4 - 2\times 12)^4 \times 10^{-4} = 104961 \text{ cm}^4$$

- Effective flexural stiffness of cross-section

Creep coefficient is similarly determined as $\varphi_t = 1.32$

$$E_{c,eff} = \frac{E_{cm}}{1 + (N_{G,Ed}/N_{Ed})\varphi_t} = \frac{41.1}{1 + (4500/11000)\times 1.32} = 26.7 \text{ GPa}$$

$$\begin{aligned}(EI)_{eff} &= E_a I_a + 0.6 E_{c,eff} I_c \\ &= \left(210\times 10^3 \times 28940\times 10^4 + 0.6\times 26.7\times 10^3 \times 104961\times 10^4\right)\times 10^{-3} \\ &= 7.76\times 10^{10} \text{ kN}\cdot\text{mm}^2\end{aligned}$$

- Confinement effect and design plastic resistance of cross-section

$$N_{cr} = \frac{\pi^2 (EI)_{eff}}{L_{eff}^2} = \frac{\pi^2 \times 7.76\times 10^{10}}{4000^2} = 47849 \text{ kN}$$

$$N_{pl,Rk} = A_a f_y + A_c f_{ck} = (14900\times 460 + 114800\times 72)\times 10^{-3} = 15121 \text{ kN}$$

$$\bar{\lambda} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} = \sqrt{\frac{15121}{47849}} = 0.562 > 0.5$$

Since the relative slenderness ratio is higher than 0.5, the confinement effect is not taken into account, thus

$$N_{pl,Rd} = A_a f_{yd} + A_c f_{cd} = \left(14900 \times 460 + 114800 \times 48\right) \times 10^{-3} = 12365 \text{ kN}$$

- Steel contribution ratio

$$\delta = A_a f_{yd} / N_{pl,Rd} = \left(14900 \times 10^{-3} \times 460\right) / 12365 = 0.554 < 0.9$$

- Buckling resistance

Buckling curve “a” is used, imperfection factor α=0.21

$$\Phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right] = 0.5\left[1 + 0.21 \times \left(0.562 - 0.2\right) + 0.562^2\right] = 0.696$$

$$\chi = \frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}} = \frac{1}{0.696 + \sqrt{0.696^2 - 0.562^2}} = 0.904$$

The axial buckling resistance is:

$$N_{b,\mathrm{Rd}} = \chi N_{\mathrm{pl,Rd}} = 0.904 \times 12365 = 11178 \text{ kN}$$

Thus, the buckling resistance is almost the same as the CHS 508x12.5 - S355 steel with C40/50 concrete infilled as calculated in Section 1.2.

- Reductions of sectional and surface area

The column section area is thus reduced by

$$\frac{\Delta A_f}{A_f} = \frac{\pi/4 \times \left(508^2 - 406.4^2\right)}{\pi/4 \times 508^2} = 36\%$$

The surface area of the column is reduced by

$$\frac{\Delta A_s}{A_s} = \frac{\pi \times \left(508 - 406.4\right)}{\pi \times 508} = 20\%$$

With the reduced surface area, the cost of fire protection material may be reduced since the labour cost for applying the fire protection material is based on the surface area. In addition, welding work and labour cost will be reduced as the less construction materials are needed for smaller column size.

1.6 Summary

For CFST columns subject to axial compression force only (mainly used in braced frames with simple construction), the use of high strength concrete will benefit more than the use of higher grade steels, compared with the increase of cost.

With the use of high strength materials, the column size is reduced. As a result, the fabrication cost of column and labour cost for applying fire protection are reduced, and the usable floor area is increased.

2 Example 2

2.1 General

In Example 2, the design resistance of a circular concrete filled steel tubular member with encased reinforcements and UC steel section is checked against combined compression and uniaxial bending moment about the major axis. The dimensions of the CFST column are shown in Figure 3.

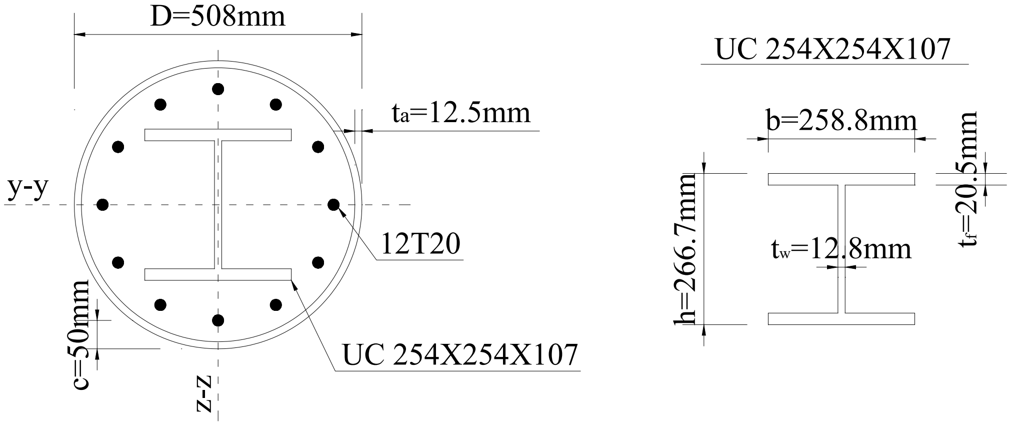
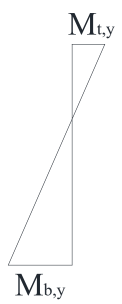


Figure 3: Cross-sectional dimensions of CFST column in Example 2

The column lengths and design loads are given as:

Column system length	L=4000 mm
Effective length	L_{eff} =4000 mm
Total design axial load	N_{Ed}=10000 kN
Design axial load that is permanent	$N_{G,Ed}$=4000 kN



Design moment at bottom around y-y axis	$M_{b,y}$=700 kN.m
Design moment at top around y-y axis	$M_{t,y}$=-500 kN.m

To evaluate the resistance, the following steel, concrete and reinforcing steel are used:

a) CHS 508x12.5 and UC 254x254x107 - S355 steel sections with C50/60 concrete and G460 reinforcements
b) CHS 508x12.5 and UC 254x254x107 - S355 steel sections with C90/105 concrete and G460 reinforcements
c) CHS 508x12.5 and UC 254x254x107 - S500 steel sections with C50/60 concrete and G460 reinforcements

2.2 CHS 508x12.5 and UC 254x254x107 - S355 steel sections with C50/60 concrete and G460 reinforcements

- Material

Concrete	C50/60, f_{ck}=50 N/mm^2
Steel tube	Grade S355, f_y=355 N/mm^2
Embedded steel section	Grade S355, f_{ek}=355 N/mm^2
Reinforcements	Grade 460, f_{sk}=460 N/mm^2

- Design strength and modulus of material

Unless otherwise stated, the subscript "e" stands for the encased steel section, and the "s" stands for the reinforcing steel.

$$f_{yd} = f_y/\gamma_a = 355/1.0 = 355\ \text{N/mm}^2$$

$$f_{sd} = f_{sk}/\gamma_s = 460/1.15 = 400\ \text{N/mm}^2$$

$$f_{ed} = f_{ek}/\gamma_a = 355/1.0 = 355\ \text{N/mm}^2$$

$$f_{cd} = f_{ck}/\gamma_c = 50/1.5 = 33.3\ \text{N/mm}^2$$

$$f_{cm} = f_{ck} + 8 = 50 + 8 = 58\ \text{N/mm}^2$$

$$E_a = E_s = E_e = 210\ \text{GPa}$$

$$E_{cm} = 22\left(f_{cm}/10\right)^{0.3} = 22\left(58/10\right)^{0.3} = 37.3\ \text{GPa}$$

- Cross sectional areas

$$A = (\pi/4)D^2 = (\pi/4)\times 508^2 = 202683\ \text{mm}^2$$

$$A_a = (\pi/4)\left[D^2 - (D-2t_a)^2\right] = (\pi/4)\left[508^2 - (508 - 2\times 12.5)^2\right] = 19458\ \text{mm}^2$$

$$A_e = bh - (b - t_w)(h - 2t_f)$$

$$= 258.8 \times 266.7 - (258.8 - 12.8)(266.7 - 2 \times 20.5) = 13500 \text{ mm}^2$$

$$A_s = 12(\pi/4)d^2 = 12 \times (\pi/4) \times 20^2 = 3770 \text{ mm}^2$$

$$A_c = A - A_a - A_e - A_s = 202683 - 19458 - 13500 - 3770 = 165955 \text{ mm}^2$$

- Second moment of areas

For simplicity, the reinforcements are equivalently converted to a circular tube based on the same cross-sectional area and position of centreline, as shown in Figure 4.

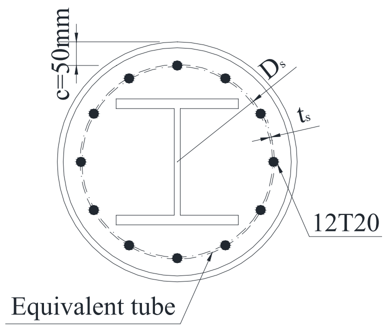


Figure 4: Equivalent tube section for reinforcing steel

$$t_s = \frac{A_s}{\pi(D - 2c)} = \frac{3770}{\pi \times (508 - 2 \times 50)} = 2.94 \text{ mm}$$

$$D_s = D - 2c + t_s = 508 - 2 \times 50 + 2.94 = 410.9 \text{ mm}$$

$$I = (\pi/64)D^4 = (\pi/64) \times 508^4 \times 10^{-4} = 326907 \text{ cm}^4$$

$$I_a = (\pi/64)\left[D^4 - (D - 2t_a)^4\right]$$

$$= (\pi/64)\left[508^4 - (508 - 2 \times 12.5)^4\right] \times 10^{-4} = 59755 \text{ cm}^4$$

$$I_s = (\pi/64)\left[D_s^4 - (D_s - 2t_s)^4\right]$$

$$= (\pi/64)\left[410.9^4 - (410.9 - 2 \times 2.94)^4\right] \times 10^{-4} = 7840 \text{ cm}^4$$

$$I_{ey} = \frac{1}{12}\left[bh^3 - (b - t_w)(h - 2t_f)^3\right]$$

$$= \frac{1}{12}\left[258.8 \times 266.7^3 - (258.8 - 12.8)(266.7 - 2 \times 20.5)^3\right] \times 10^{-4} = 17343 \text{ cm}^4$$

$$I_{ez}=\frac{1}{12}\left[2t_f b^3+\left(h-2t_f\right)t_w^3\right]$$

$$=\frac{1}{12}\left[2\times20.5\times258.8^3+\left(266.7-2\times20.5\right)\times12.8^3\right]\times10^{-4}=5926\ \text{cm}^4$$

$$I_{cy}=I-I_a-I_s-I_{ey}=326907-59755-7840-17343=241969\ \text{cm}^4$$

$$I_{cz}=I-I_a-I_s-I_{ez}=326907-59755-7840-5926=253386\ \text{cm}^4$$

- Plastic modulus

$$W=D^3/6=508^3/6\times10^{-3}=21850\ \text{cm}^3$$

$$W_a=\left[D^3-\left(D-2t_a\right)^3\right]/6=\left[508^3-\left(508-2\times12.5\right)^3\right]/6\times10^{-3}=3070\ \text{cm}^3$$

$$W_s=\left[D_s^3-\left(D_s-2t_s\right)^3\right]/6=\left[410.9^3-\left(410.9-2\times2.94\right)^3\right]/6\times10^{-3}=489\ \text{cm}^3$$

$$W_{ey}=\frac{1}{4}\left[bh^2-\left(b-t_w\right)\left(h-2t_f\right)^2\right]$$

$$=\frac{1}{4}\left[258.8\times266.7^2-\left(258.8-12.8\right)\left(266.7-2\times20.5\right)^2\right]\times10^{-3}=1469\ \text{cm}^3$$

$$W_{ez}=\frac{1}{4}\left[2t_f b^2+\left(h-2t_f\right)t_w^2\right]$$

$$=\frac{1}{4}\left[2\times20.5\times258.8^2+\left(266.7-2\times20.5\right)\times12.8^2\right]\times10^{-4}=696\ \text{cm}^4$$

$$W_{cy}=W-W_a-W_s-W_{ey}=21850-3070-489-1469=16821\ \text{cm}^4$$

$$W_{cz}=W-W_a-W_s-W_{ez}=21850-3070-489-696=17594\ \text{cm}^4$$

- Check for local buckling

$$D/t_a=508/12.5=40.6<90\left(235/f_y\right)=90\left(235/355\right)=59.6$$

Resistance against local buckling is adequate!

- Long-term effect

Age of concrete at loading in days: $t_0=30$

Age of concrete at moment considered in days: $t=\infty$

Relative humidity of ambient environment: RH=50%

Perimeter of concrete section: $u=\pi\left(D-2t_a\right)=\pi\left(508\text{-}2\times12.5\right)=1517$ mm

Notional size of concrete section: $h_0=2\mathrm{A_c}/u=2\times165955/1517=219$ mm

Coefficient: $\alpha_1=\left(35/f_{cm}\right)^{0.7}=\left(35/58\right)^{0.7}=0.70$

Coefficient: $\alpha_2=\left(35/f_{cm}\right)^{0.2}=\left(35/58\right)^{0.2}=0.90$

Coefficient: $\alpha_3=\left(35/f_{cm}\right)^{0.5}=\left(35/58\right)^{0.5}=0.78$

Factor: $\varphi_{\mathrm{RH}}=\left(1+\dfrac{1-RH/100}{0.1\sqrt[3]{h_0}}\alpha_1\right)\alpha_2=\left(1+\dfrac{1-50/100}{0.1\sqrt[3]{219}}\times0.70\right)\times0.90=1.43$

Factor: $\beta\left(f_{cm}\right)=16.8/\sqrt{f_{cm}}=16.8/\sqrt{58}=2.21$

Factor: $\beta\left(t_0\right)=1/\left(0.1+t_0^{0.2}\right)=1/\left(0.1+30^{0.2}\right)=0.48$

Factor: $\varphi_0=\varphi_{\mathrm{RH}}\beta\left(f_{cm}\right)\beta\left(t_0\right)=1.43\times2.21\times0.48=1.52$

Factor: $\beta_H=1.5\left[1+\left(0.012RH\right)^{18}\right]h_0+250\alpha_3$

$=1.5\times\left[1+\left(0.012\times50\right)^{18}\right]\times219+250\times0.78=522$

Factor: $\beta_c\left(t,t_0\right)=\left(\dfrac{t-t_0}{\beta_H+t-t_0}\right)^{0.3}=\left(\dfrac{\infty-14}{522+\infty-14}\right)^{0.3}=1.0$

Creep coefficient: $\varphi_t=\varphi_0\beta_c\left(t,t_0\right)=1.52\times1.0=1.52$

- Elastic modulus of concrete considering long-term effect

(1) $E_{c,eff}=E_{cm}\dfrac{1}{1+\left(N_{G,Ed}/N_{Ed}\right)\varphi_t}=\dfrac{37.3}{1+\left(4000/10000\right)\times1.52}=23.2\ \mathrm{GPa}$

- Effective flexural stiffness of cross-section

$\left(EI\right)_{eff,y}=E_aI_a+E_sI_s+E_eI_{ey}+0.6E_{c,eff}I_{cy}$

$=\left[210\times\left(59755+7840+17343\right)+0.6\times23.2\times241969\right]\times10^4$

$=2.12\times10^{11}\ \mathrm{kN\cdot mm^2}$

$\left(EI\right)_{eff,z}=E_aI_a+E_sI_s+E_eI_{ez}+0.6E_{c,eff}I_{cz}$

$=\left[210\times\left(59755+7840+5926\right)+0.6\times23.2\times253386\right]\times10^4$

$=1.90\times10^{11}\ \mathrm{kN\cdot mm^2}$

- Elastic critical Euler buckling resistance

$$N_{cr,y} = \frac{\pi^2 (EI)_{eff,y}}{L_{eff}^2} = \frac{\pi^2 \times 2.12 \times 10^{11}}{4000^2} = 130793 \text{ kN}$$

$$N_{cr,z} = \frac{\pi^2 (EI)_{eff,z}}{L_{eff}^2} = \frac{\pi^2 \times 1.90 \times 10^{11}}{4000^2} = 116984 \text{ kN}$$

- Characteristic plastic resistance of cross-section

$$N_{pl,Rk} = A_a f_y + A_s f_{sk} + A_e f_{ek} + A_c f_{ck}$$

$$= \left[(19458 + 13500) \times 355 + 3770 \times 460 + 165955 \times 50\right] \times 10^{-3} = 21475 \text{ kN}$$

- Relative slenderness ratio

$$\overline{\lambda}_y = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{21475}{130793}} = 0.408 < 0.5$$

$$\overline{\lambda}_z = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{21475}{116984}} = 0.431 < 0.5$$

- Buckling curves and buckling reduction factors

Since a steel section is embedded in the CFST column, the buckling curves about both axis are "b". Thus, the imperfection factor is $\alpha = 0.34$.

$$\overline{\lambda} = \max\left(\overline{\lambda}_y, \overline{\lambda}_z\right) = 0.431$$

$$\Phi = 0.5\left[1 + \alpha\left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right] = 0.5\left[1 + 0.34 \times (0.431 - 0.2) + 0.431^2\right] = 0.632$$

$$\chi = \min\left(\frac{1}{\Phi + \sqrt{\Phi^2 - \overline{\lambda}^2}}, 1.0\right) = \min\left(\frac{1}{0.632 + \sqrt{0.632^2 - 0.431^2}}, 1.0\right) = 0.914$$

- Load eccentricity

$$e_{N,y} = \frac{\max\left(\left|M_{t,y}\right|, \left|M_{b,y}\right|\right)}{N_{Ed}} = \frac{700}{10000} \times 10^3 = 70 \text{ mm} > 0.1D = 50.8 \text{ mm}$$

Since the eccentricity $e_{N,y}$ is larger than 0.1D, the confinement effect is not taken into account.

- Simplified Interaction Curve

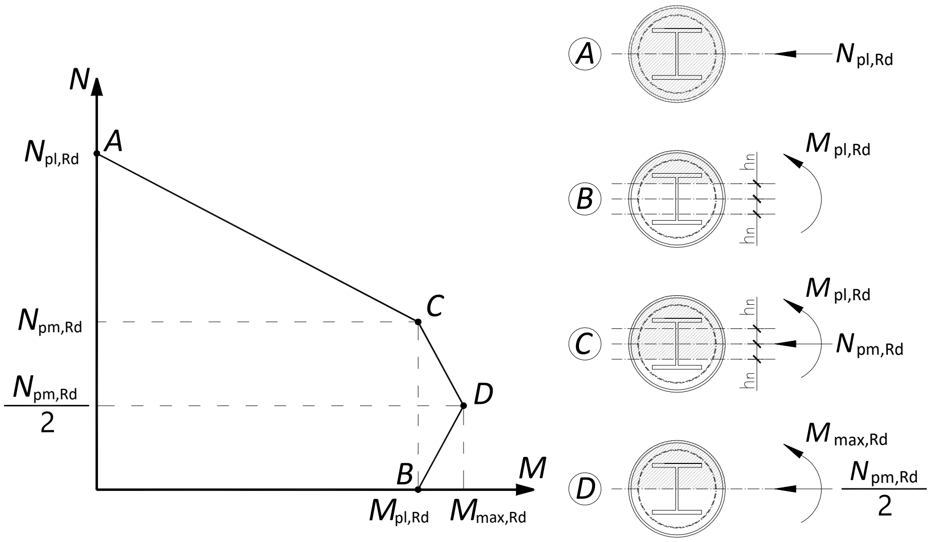


Figure 5: Simplified interaction curve for circular CFST column with encased steel section

1) Point A (0, $N_{pl,Rd}$):

Full cross-section is under uniform compression. No bending moment is resultant from the compressive stresses on the cross-section.

$$N_{pl,Rd} = A_a f_{yd} + A_s f_{sd} + A_e f_{ed} + A_c f_{cd}$$

$$= \left[(19458 + 13500) \times 355 + 3770 \times 400 + 165955 \times 33.3 \right] \times 10^{-3}$$

$$= 18753 \text{ kN}$$

2) Point B ($M_{pl,Rd}$, 0):

The cross-section is under partial compression and no axial force is formed. Assuming the neutral axis lies in the web of encased section ($h_n \leq h/2 - t_f$), the height of neutral axis is calculated where the areas of steel tube, equivalent tube for reinforcements, and concrete in the height of $2h_n$ are approximated as rectangles, and based on the force equilibrium between the tensile capacity of steel sections within the height of $2h_n$ is equal to the compression resistance of concrete in the compression zone. Unless otherwise stated, the tensile resistance of concrete in the tension zone is conservatively ignored.

$$h_n = \frac{A_c f_{cd}}{2Df_{cd} + 4t_a\left(2f_{yd} - f_{cd}\right) + 4t_s\left(2f_{sd} - f_{cd}\right) + 2t_w\left(2f_{ed} - f_{cd}\right)}$$

$$= \frac{165955 \times 33.3}{2\times508\times33.3 + 4\times12.5\times\left(2\times355 - 33.3\right) + 4\times2.94\times\left(2\times400 - 33.3\right) + 2\times12.8\times\left(2\times355 - 33.3\right)}$$

$$= 58.8 \text{ mm}$$

$$h_n = 58.8 \text{ mm} < h/2 - t_f = 266.7/2 - 20.5 = 112.85 \text{ mm}$$

Thus, the neutral axial lies in the web of the encased section. The plastic modulus of steel tube, equivalent tube of reinforcements, encased section, and concrete in the height of $2h_n$, bending about centreline of the cross-section are calculated as:

$$W_{a,n} = 2t_a h_n^2 = 2\times12.5\times58.8^2\times10^{-3} = 86.4 \text{ cm}^3$$

$$W_{s,n} = 2t_s h_n^2 = 2\times2.94\times58.8^2\times10^{-3} = 20.3 \text{ cm}^3$$

$$W_{ey,n} = t_w h_n^2 = 12.8\times58.8^2\times10^{-3} = 44.3 \text{ cm}^3$$

$$W_{cy,n} = \left(D - 2t_a - 2t_s - t_w\right)h_n^2 = \left(508 - 2\times12.5 - 2\times2.94 - 12.8\right)\times58.8^2\times10^{-3} = 1605 \text{ cm}^3$$

Taking moment about the centreline of the cross-section, the plastic bending resistance is determined from:

$$M_{pl,Rd} = \left(W_a - W_{a,n}\right)f_{yd} + \left(W_s - W_{s,n}\right)f_{sd} + \left(W_{ey} - W_{ey,n}\right)f_{ed} + 0.5\left(W_{cy} - W_{cy,n}\right)f_{cd}$$

$$= \left[\left(3070 - 86.4\right)\times355 + \left(489 - 20.3\right)\times400 + \left(1469 - 44.3\right)\times355 + 0.5\times\left(16821 - 1605\right)\times33.3\right]\times10^{-3}$$

$$= 2006 \text{ kN}\cdot\text{m}$$

It should be noted that the plastic bending resistance can be calculated by taking moment about either line on the cross-section parallel to the y-y axis, as long as the aforementioned plastic modulus are determined according to the referred line.

3) Point C ($M_{pl,Rd}$, $N_{pm,Rd}$):

The cross-section is under partial compression but axial force is resultant from the compressive stresses. The axial force is equal to the compression capacities of concrete in the compression zone and steel sections within the height of $2h_n$. It is mentioned above that the compression capacity of steel sections within the height of $2h_n$ is equal to the compression capacity of concrete in the compression zone and out of the height of $2h_n$.

Thus, the axial force is actually the full cross-sectional compression capacity of concrete and determined from:

$$N_{pm,Rd} = A_c f_{cd} = 165955 \times 33.3 \times 10^{-3} = 5526 \text{ kN}$$

4) Point D ($M_{\max,\mathrm{Rd}}$, $N_{\mathrm{pm,Rd}}/2$):

The maximum plastic moment resistance $M_{\max,\mathrm{Rd}}$ is calculated when the h_n is equal to 0.

$$M_{\max,Rd} = W_a f_{yd} + W_s f_{sd} + W_{ey} f_{ed} + 0.5 W_{cy} f_{cd}$$

$$= \left[3070 \times 355 + 489 \times 400 + 1469 \times 355 + 0.5 \times 16821 \times 33.3\right] \times 10^{-3} = 2087 \text{ kN} \cdot \text{m}$$

- Steel contribution ratio

$$\delta = \left(A_a f_{yd} + A_e f_{ed}\right) / N_{pl,Rd}$$

$$= \left(19458 + 13500\right) \times 355 \times 10^{-3} / 18753 = 0.625 < 0.9$$

- Check for resistance of column in axial compression

$$\frac{N_{Ed}}{\chi N_{pl,\mathrm{R}d}} = \frac{10000}{0.914 \times 18753} = \frac{10000}{17140} = 0.583 < 1.0$$

Thus, the buckling resistance under axial compression is adequate!

- Effective flexural stiffness considering long-term effect

$$\left(EI\right)_{eff,II} = K_0 \left(E_a I_a + E_s I_s + E_e I_{ey} + K_{e,II} E_{c,\mathrm{m}} I_{cy}\right)$$

$$= 0.9 \times \left[210 \times \left(59755 + 7840 + 17343\right) + 0.5 \times 23.2 \times 241969\right] \times 10^4$$

$$= 1.86 \times 10^{11} \text{ kN} \cdot \text{mm}^2$$

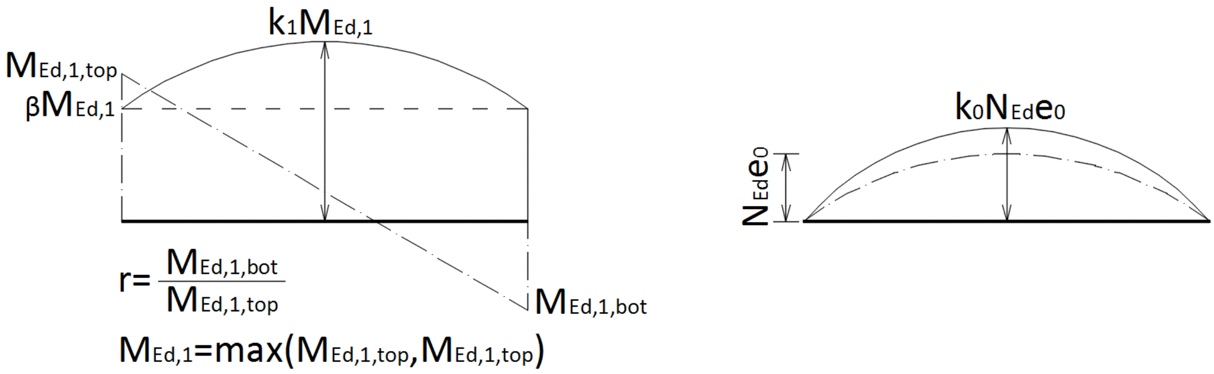
- Critical normal force about y-y axis

The effective length is taken as the system length of column.

$$N_{cr,\mathrm{eff}} = \frac{\pi^2 \left(EI\right)_{eff,\mathrm{II}}}{L^2} = \frac{\pi^2 \times 1.86 \times 10^{11}}{4000^2} = 114601 \text{ kN}$$

- Second-order effect (refer to Clause 3.3.2 (2) of BC4)

The second-order effect should be considered for both moments from first-order analysis and moment from imperfection as shown in Figure 6.



(a) Moment from first-order analysis (b) Moment from imperfection

Figure 6: Consideration for second order effect

According to buckling curve “b”, the initial imperfection about y-y axial is:

$$e_{0,y} = L/200 = 4000/200 = 20 \text{ mm}$$

Accordingly, the bending moment by the initial imperfection is determined as:

$$M_0 = N_{Ed} e_{0,y} = 10000 \times 20/1000 = 200 \text{ kN} \cdot \text{m}$$

According to the moment diagram by the initial imperfection, the factor β_0 for determination of moment to second-order effect is equal to 1.0. Thus, the amplification factor for the moment by the imperfection is calculated from:

$$k_0 = \frac{\beta_0}{1 - N_{Ed}/N_{cr,\text{eff}}} = \frac{1.0}{1 - 10000/114601} = 1.096$$

According to the first-order design moment diagram, the ratio of end moments is calculated as:

$$r = M_{t,y}/M_{b,y} = -500/700 = -0.714$$

Thus, the factor β_1 for determination of moment to second-order effect is determined:

$$\beta_1 = \max\left(0.66 + 0.44r, 0.44\right) = \max\left(0.66 + 0.44 \times \left(-0.714\right), 0.44\right) = 0.44$$

Thus, the amplification factor for the moment by the imperfection is calculated from:

$$k_1 = \frac{\beta_1}{1 - N_{Ed}/N_{cr,\text{eff}}} = \frac{0.44}{1 - 10000/114601} = 0.482$$

Thus, the design moment, considering second-order effect, is calculated as:

$$M_{Ed} = Max\left[k_0 M_0 + k_1 Max\left(\left|M_{t,y}\right|, \left|M_{b,y}\right|\right), Max\left(\left|M_{t,y}\right|, \left|M_{b,y}\right|\right)\right]$$
$$= Max\left[1.096 \times 200 + 0.482 \times Max\left(\left|-500\right|, \left|700\right|\right), Max\left(\left|-500\right|, \left|700\right|\right)\right] = 700 \text{ kN} \cdot \text{m}$$

In this case, the second-order effect is not significant and the maximum end moment is taken as the design moment.

- Check for resistance of column in combined compression and uniaxial bending

For $N_{Ed} > N_{pm,Rd} = 5526\ kN$, the value for determining the plastic bending resistance $M_{pl,N,Rd}$ taking into account the normal force N_{Ed} is calculated from:

$$\mu_d = \frac{N_{pl,Rd} - N_{Ed}}{N_{pl,Rd} - N_{pm,Rd}} = \frac{18753 - 10000}{18753 - 5526} = 0.662$$ (refer to Eq.(3.24) of BC4)

$$\frac{M_{Ed}}{M_{pl,N,Rd}} = \frac{M_{Ed}}{\mu_d M_{pl,Rd}} = \frac{700}{0.662 \times 2006} = 0.527 < \alpha_M = 0.9$$

Thus, the resistance for combined axial compression and uniaxial bending is adequate. The external design force and bending moment, and the design M-N interaction curve are plotted in Figure 7.

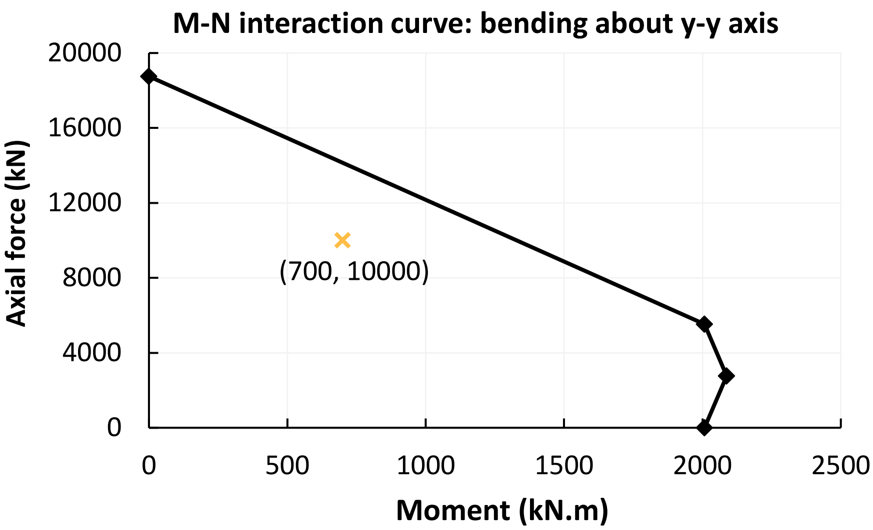


Figure 7: Design M-N interaction curve of circular CFST column

2.3 CHS 508x12.5 and UC 254x254x107 - S355 steel sections with C90/105 concrete and G460 reinforcements

In this section, the normal strength concrete (NSC) C40/50 is replaced by high strength concrete (HSC) C90/105. The steel grade is not changed.

- Design strength

Effective compressive strength and modulus of elasticity of the HSC are taken from Table 2.2, Eq.(2.2) and Eq.(2.3) of BC4.

$$f_{ck} = 72\ \text{N/mm}^2;\ \ E_{cm} = 41.1\ \text{GPa}$$

$$f_{cd} = f_{ck}/\gamma_c = 72/1.5 = 48\ \text{N/mm}^2$$

$$f_{cm} = f_{ck} + 8 = 72 + 8 = 80\ \text{N/mm}^2$$

- Effective flexural stiffness of cross-section

Creep coefficient could be similarly determined as $\varphi_t = 1.12$

$$E_{c,eff} = \frac{E_{cm}}{1+\left(N_{G,Ed}/N_{Ed}\right)\varphi_t} = \frac{41.1}{1+\left(4000/10000\right)\times 1.12} = 28.3\ \text{GPa}$$

$$(EI)_{eff,y} = E_a I_a + E_s I_s + E_e I_{ey} + 0.6 E_{c,eff} I_{cy}$$
$$= \left[210\times\left(59755+7840+17343\right)+0.6\times 28.3\times 241969\right]\times 10^4$$
$$= 2.19\times 10^{11}\ \text{kN}\cdot\text{mm}^2$$

$$(EI)_{eff,z} = E_a I_a + E_s I_s + E_e I_{ez} + 0.6 E_{c,eff} I_{cz}$$
$$= \left[210\times\left(59755+7840+5926\right)+0.6\times 28.3\times 253386\right]\times 10^4$$
$$= 1.97\times 10^{11}\ \text{kN}\cdot\text{mm}^2$$

- Characteristic plastic resistance of cross-section

$$N_{cr,y} = \frac{\pi^2 (EI)_{eff,y}}{L_{eff}^2} = \frac{\pi^2\times 2.19\times 10^{11}}{4000^2} = 135397\ \text{kN}$$

$$N_{cr,z} = \frac{\pi^2 (EI)_{eff,z}}{L_{eff}^2} = \frac{\pi^2\times 1.97\times 10^{11}}{4000^2} = 121805\ \text{kN}$$

$$N_{pl,Rk} = A_a f_y + A_s f_{sk} + A_e f_{ek} + A_c f_{ck}$$

$$= \left[(19458 + 13500) \times 355 + 3770 \times 460 + 165955 \times 90 \right] \times 10^{-3} = 25395 \text{ kN}$$

- Relative slenderness ratio, buckling curves and buckling reduction factors

$$\overline{\lambda_y} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{25395}{135397}} = 0.433 < 0.5 \; ; \overline{\lambda_z} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{25395}{121805}} = 0.457 < 0.5$$

For buckling curves about both axis "b", the imperfection factor is $\alpha = 0.34$

$$\overline{\lambda} = \max\left(\overline{\lambda_y}, \overline{\lambda_z}\right) = 0.457$$

$$\Phi = 0.5\left[1 + \alpha\left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right] = 0.5\left[1 + 0.34 \times (0.457 - 0.2) + 0.457^2\right] = 0.648$$

$$\chi = \min\left(\frac{1}{\Phi + \sqrt{\Phi^2 + \overline{\lambda}^2}}, 1.0\right) = \min\left(\frac{1}{0.648 + \sqrt{0.648^2 - 0.457^2}}, 1.0\right) = 0.903$$

The confinement effect is also ignored since the eccentricity is larger than 0.1D.

- M-N interaction curve

Point A (0, $N_{\text{pl,Rd}}$):

$$N_{pl,Rd} = A_a f_{yd} + A_s f_{sd} + A_e f_{ed} + A_c f_{cd}$$

$$= \left[(19458 + 13500) \times 355 + 3770 \times 400 + 165955 \times 48 \right] \times 10^{-3} = 21187 \text{ kN}$$

Buckling resistance: $\chi N_{pl,Rd} = 0.903 \times 21187 = 19132 \text{ kN}$

Point B ($M_{\text{pl,Rd}}$, 0):

$$h_n = \frac{A_c f_{cd}}{2Df_{cd} + 4t_a\left(2f_{yd} - f_{cd}\right) + 4t_s\left(2f_{sd} - f_{cd}\right) + 2t_w\left(2f_{ed} - f_{cd}\right)}$$

$$= \frac{165955 \times 48}{2 \times 508 \times 48 + 4 \times 12.5 \times (2 \times 355 - 48) + 4 \times 2.94 \times (2 \times 400 - 48) + 2 \times 12.8 \times (2 \times 355 - 48)}$$

$$= 73.7 \text{ mm}$$

$h_n = 73.7 \text{ mm} < h/2 - t_f = 266.7/2 - 20.5 = 112.85 \text{ mm}$, thus, the neutral axial also lies in the web of the encased section.

$$W_{a,n} = 2t_a h_n^2 = 2\times 12.5\times 73.7^2\times 10^{-3} = 135.8\ \text{cm}^3$$

$$W_{s,n} = 2t_s h_n^2 = 2\times 2.94\times 73.7^2\times 10^{-3} = 31.9\ \text{cm}^3$$

$$W_{ey,n} = t_w h_n^2 = 12.8\times 73.7^2\times 10^{-3} = 69.5\ \text{cm}^3$$

$$W_{cy,n} = \left(D - 2t_a - 2t_s - t_w\right) h_n^2 = \left(508 - 2\times 12.5 - 2\times 2.94 - 12.8\right)\times 73.7^2\times 10^{-3} = 2522\ \text{cm}^3$$

$$M_{pl,Rd} = \left(W_a - W_{a,n}\right) f_{yd} + \left(W_s - W_{s,n}\right) f_{sd} + \left(W_{ey} - W_{ey,n}\right) f_{ed} + 0.5\left(W_{cy} - W_{cy,n}\right) f_{cd}$$

$$= \left[\left(3070 - 135.8\right)\times 355 + \left(489 - 31.9\right)\times 400 + \left(1469 - 65.9\right)\times 355 + 0.5\times\left(16821 - 2522\right)\times 33.3\right]\times 10^{-3}$$

$$= 2065\ \text{kN}\cdot\text{m}$$

Point C ($M_{\text{pl,Rd}}$, $N_{\text{pm,Rd}}$):

$$N_{pm,Rd} = A_c f_{cd} = 165955\times 48\times 10^{-3} = 7964\ \text{kN}$$

Point D ($M_{\text{max,Rd}}$, $N_{\text{pm,Rd}}/2$):

$$M_{\max,Rd} = W_a f_{yd} + W_s f_{sd} + W_{ey} f_{ed} + 0.5 W_{cy} f_{cd}$$

$$= \left[3070\times 355 + 489\times 400 + 1469\times 355 + 0.5\times 16821\times 48\right]\times 10^{-3}$$

$$= 2211\ \text{kN}\cdot\text{m}$$

- Steel contribution ratio

$$\delta = \left(A_a f_{yd} + A_e f_{ed}\right) / N_{pl,Rd}$$

$$= \left(19458 + 13500\right)\times 355\times 10^{-3} / 21187 = 0.552 < 0.9$$

2.4 CHS 508x12.5 and UC 254x254x107 - S500 steel sections with C50/60 concrete and G460 reinforcements

In this section, the mild steel S355 is replaced by the high tensile steel (HTS) S500, the concrete grade is not changed.

- Characteristic plastic resistance of cross-section

$$N_{pl,Rk} = A_a f_y + A_s f_{sk} + A_e f_{ek} + A_c f_{ck}$$

$$= \left[\left(19458 + 13500\right)\times 500 + 3770\times 460 + 165955\times 50\right]\times 10^{-3} = 26530\ \text{kN}$$

- Relative slenderness ratio, buckling curves and buckling reduction factors

$$\overline{\lambda}_y = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{26530}{130793}} = 0.450 < 0.5$$

$$\overline{\lambda_z} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{26530}{116984}} = 0.476 < 0.5$$

For buckling curves about both axis "b", the imperfection factor is $\alpha = 0.34$

$$\overline{\lambda} = \max\left(\overline{\lambda_y}, \overline{\lambda_z}\right) = 0.476$$

$$\Phi = 0.5\left[1 + \alpha\left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right] = 0.5\left[1 + 0.34 \times (0.476 - 0.2) + 0.476^2\right] = 0.660$$

$$\chi = \min\left(\frac{1}{\Phi + \sqrt{\Phi^2 + \overline{\lambda}^2}}, 1.0\right) = \min\left(\frac{1}{0.660 + \sqrt{0.660^2 - 0.476^2}}, 1.0\right) = 0.895$$

The confinement effect is also ignored since the eccentricity is larger than 0.1D.

- M-N interaction curve

Point A (0, $N_{pl,Rd}$):

$$N_{pl,Rd} = A_a f_{yd} + A_s f_{sd} + A_e f_{ed} + A_c f_{cd}$$
$$= \left[(19458 + 13500) \times 500 + 3770 \times 400 + 165955 \times 33.3\right] \times 10^{-3} = 23538 \text{ kN}$$

Buckling resistance: $\chi N_{pl,Rd} = 0.895 \times 23538 = 21067 \text{ kN}$

Point B ($M_{pl,Rd}$, 0):

$$h_n = \frac{A_c f_{cd}}{2Df_{cd} + 4t_a\left(2f_{yd} - f_{cd}\right) + 4t_s\left(2f_{sd} - f_{cd}\right) + 2t_w\left(2f_{ed} - f_{cd}\right)}$$
$$= \frac{165955 \times 33.3}{2 \times 508 \times 33.3 + 4 \times 12.5 \times (2 \times 500 - 33.3) + 4 \times 2.94 \times (2 \times 400 - 33.3) + 2 \times 12.8 \times (2 \times 500 - 33.3)}$$
$$= 47.3 \text{ mm}$$

The neutral axial also lies in the web of the encased section.

$$W_{a,n} = 2t_a h_n^2 = 2 \times 12.5 \times 47.3^2 \times 10^{-3} = 55.9 \text{ cm}^3$$

$$W_{s,n} = 2t_s h_n^2 = 2 \times 2.94 \times 47.3^2 \times 10^{-3} = 13.2 \text{ cm}^3$$

$$W_{ey,n} = t_w h_n^2 = 12.8 \times 47.3^2 \times 10^{-3} = 28.6 \text{ cm}^3$$

$$W_{cy,n} = \left(D - 2t_a - 2t_s - t_w\right) h_n^2 = (508 - 2 \times 12.5 - 2 \times 2.94 - 12.8) \times 47.3^2 \times 10^{-3} = 1039 \text{ cm}^3$$

$$M_{pl,Rd} = \left(W_a - W_{a,n}\right) f_{yd} + \left(W_s - W_{s,n}\right) f_{sd} + \left(W_{ey} - W_{ey,n}\right) f_{ed} + 0.5\left(W_{cy} - W_{cy,n}\right) f_{cd}$$

$$= \left[\left(3070 - 55.9\right) \times 500 + \left(489 - 13.2\right) \times 400 + \left(1469 - 28.6\right) \times 500 + 0.5 \times \left(16821 - 1039\right) \times 33.3\right] \times 10^{-3}$$

$$= 2681 \text{ kN} \cdot \text{m}$$

Point C ($M_{pl,Rd}$, $N_{pm,Rd}$):

$$N_{pm,Rd} = A_c f_{cd} = 165955 \times 33.3 \times 10^{-3} = 5530 \text{ kN}$$

Point D ($M_{max,Rd}$, $N_{pm,Rd}/2$):

$$M_{max,Rd} = W_a f_{yd} + W_s f_{sd} + W_{ey} f_{ed} + 0.5 W_{cy} f_{cd}$$

$$= \left[3070 \times 500 + 489 \times 400 + 1469 \times 500 + 0.5 \times 16821 \times 33.3\right] \times 10^{-3}$$

$$= 2746 \text{ kN} \cdot \text{m}$$

- Steel contribution ratio:

$$\delta = \left(A_a f_{yd} + A_e f_{ed}\right) / N_{pl,Rd}$$

$$= \left(19458 + 13500\right) \times 500 \times 10^{-3} / 23538 = 0.701 < 0.9$$

2.5 Comparison and summary

The design resistances are compared for the aforementioned three composite sections. The composite section with steel tube of S355, encased steel section of S355, concrete of C50/60, and reinforcing steel of G460 is referred to for the comparison.

It can been seen in, the axial buckling resistance ($\chi N_{pl,Rd}$) of the CFST column is improved by 11.6 % by use of high strength concrete C90/105 replacing normal strength concrete C50/60. However the increase of moment resistances ($M_{pl,Rd}$ and $M_{max,Rd}$) are smaller (less than 6%).

By use of steel S500 replacing S355, the axial buckling resistance is improved by 22.9%, and the increase of moment resistance is higher than 30%.

Table 1: Comparisons between circular CFST columns with various material strengths

Material grades (Steel+ Concrete+ Rebars)	Steel contribution ratios	Design resistances			
		$\chi N_{pl,Rd}$	$N_{pm,Rd}$	$M_{pl,Rd}$	$M_{max,Rd}$
S355+C50/60+G460	0.625	0	0	0	0
S355+C90/105+G460	0.552	11.6%	44.0%	2.9%	5.9%
S500+C50/60+G460	0.701	22.9%	0	33.6%	31.5%

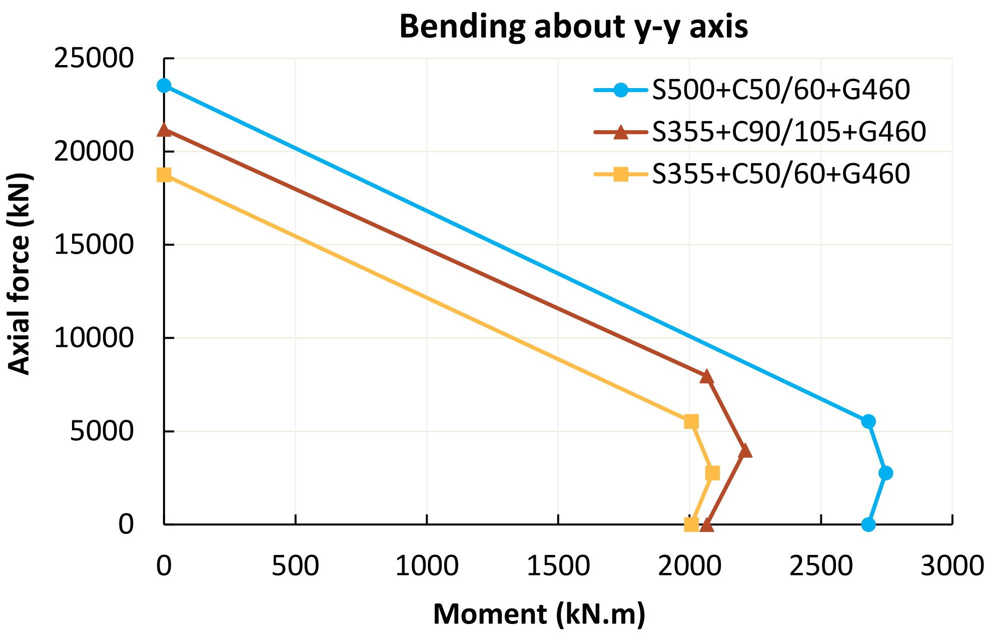


Figure 8: Design M-N interaction curves for CFST columns with various material grades

3 Example 3

3.1 General

In Example 3, the design resistance of a rectangular concrete filled steel tubular column are checked against combined axial compression and bi-axial bending moments. The dimensions of the rectangular CFST column are shown in Figure 9.

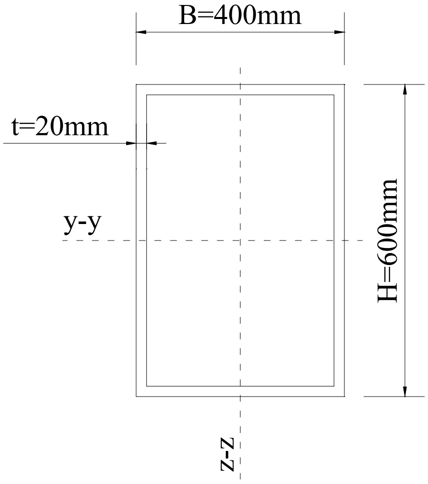
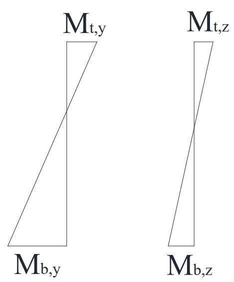


Figure 9: Cross-sectional dimensions of CFST column in Example 3

The column lengths and design loads are given as:

Column system length	L=6000 mm
Effective length	L_{eff} =6000 mm
Total design axial load	N_{Ed}=12000 kN
Design axial load that is permanent	$N_{G,Ed}$=5000 kN
Design moment at bottom around y-y axis	$M_{b,y}$=900 kN.m
Design moment at top around y-y axis	$M_{t,y}$=-550 kN.m



Design moment at bottom around y-y axis	$M_{b,z}$=-600 kN.m
Design moment at top around y-y axis	$M_{t,z}$=350 kN.m

To evaluate and compare their resistance, the following steel and concrete material grades are taken into account:

a) RHS 400x600x20 - S355 steel with C50/60 concrete

b) RHS 400x600x20 - S355 steel with C90/105 concrete

c) RHS 400x600x20 - S500 steel with C50/60 concrete

d) RHS 400x600x20 - S500 steel with C90/105 concrete

3.2 RHS 400x600x20 - S355 steel tube infilled with C50/60 concrete

- Material

Concrete	C50/60, f_{ck}=50 N/mm^2
Steel tube	Grade S355, f_y=355 N/mm^2

- Design strengths and modulus of material

$$f_{yd} = f_y/\gamma_a = 355/1.0 = 355 \text{ N/mm}^2$$

$$f_{cd} = f_{ck}/\gamma_c = 50/1.5 = 33.3 \text{ N/mm}^2$$

$$f_{cm} = f_{ck} + 8 = 50 + 8 = 58 \text{ N/mm}^2$$

$$E_a = 210 \text{ GPa}$$

$$E_{cm} = 22\left(f_{cm}/10\right)^{0.3} = 22\left(58/10\right)^{0.3} = 37.3 \text{ GPa}$$

- Cross sectional areas

$$A = BH = 400 \times 600 \times 10^{-2} = 2400 \text{ cm}^2$$

$$A_a = \left[BH - (B - 2t_a)(H - 2t_a)\right] = \left[400 \times 600 - (400 - 2 \times 20)(600 - 2 \times 20)\right] \times 10^{-2} = 384 \text{ cm}^2$$

$$A_c = A - A_a = 240000 - 38400 \times 10^{-2} = 2016 \text{ cm}^2$$

- Second moment of areas

$$I_y = BH^3/12 = 400 \times 600^3/12 \times 10^{-4} = 720000 \text{ cm}^4$$

$$I_z = HB^3/12 = 600 \times 400^3/12 \times 10^{-4} = 320000 \text{ cm}^4$$

$$I_{ay} = \left[BH^3 - (B - 2t_a)(H - 2t_a)^3\right]/12 = \left[400 \times 600^3 - (400 - 2 \times 20)(600 - 2 \times 20)^3\right]/12 \times 10^{-4}$$

$$= 193152 \text{ cm}^4$$

$$I_{az}=\left[HB^3-(H-2t_a)(B-2t_a)^3\right]/12=\left[600\times400^3-(600-2\times20)(400-2\times20)^3\right]/12\times10^{-4}$$

$$=102272 \text{ cm}^4$$

$$I_{cy}=I_y-I_{ay}=720000-193152=526848 \text{ cm}^4$$

$$I_{cz}=I_z-I_{az}=320000-193152=217728 \text{ cm}^4$$

- Plastic modulus

$$W_y=BH^2/4=400\times600^2/4\times10^{-3}=36000 \text{ cm}^3$$

$$W_z=HB^2/4=600\times400^2/4\times10^{-3}=24000 \text{ cm}^3$$

$$W_{ay}=\left[BH^2-(B-2t_a)(H-2t_a)^2\right]/4=\left[400\times600^2-(400-2\times20)(600-2\times20)^2\right]/4\times10^{-3}$$

$$=7776 \text{ cm}^3$$

$$W_{az}=\left[HB^2-(H-2t_a)(B-2t_a)^2\right]/4=\left[600\times400^2-(600-2\times20)(400-2\times20)^2\right]/4\times10^{-3}$$

$$=5856 \text{ cm}^3$$

$$W_{cy}=W_y-W_{ay}=36000-7776=28224 \text{ cm}^3$$

$$W_{cz}=W_z-W_{az}=24000-5856=18144 \text{ cm}^3$$

- Check for local buckling

$$H/t_a=600/20=30<52\left(235/f_y\right)=52\left(235/355\right)=42.3$$

Resistance against local buckling is adequate!

- Long-term effect

Assuming age of concrete at loading in days: $t_0=30$

Age of concrete at moment considered in days: $t=\infty$

Relative humidity of ambient environment: RH=50%

Perimeter of concrete section: $u=2\left(B-2t_a\right)+2\left(H-2t_a\right)$

$$=2\left(400-2\times20\right)+2\left(600-2\times20\right)=1840 \text{ mm}$$

Notional size of concrete section: $h_0=2A_c/u=2\times201600/1840=219 \text{ mm}$

Coefficient: $\alpha_1 = (35/f_{cm})^{0.7} = (35/58)^{0.7} = 0.70$

Coefficient: $\alpha_2 = (35/f_{cm})^{0.2} = (35/58)^{0.2} = 0.90$

Coefficient: $\alpha_3 = (35/f_{cm})^{0.5} = (35/58)^{0.5} = 0.78$

Factor: $\varphi_{\mathrm{RH}} = \left(1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}}\alpha_1\right)\alpha_2 = \left(1 + \frac{1 - 50/100}{0.1\sqrt[3]{219}} \times 0.70\right) \times 0.90 = 1.43$

Factor: $\beta(f_{cm}) = 16.8/\sqrt{f_{cm}} = 16.8/\sqrt{58} = 2.21$

Factor: $\beta(t_0) = 1/(0.1 + t_0^{0.2}) = 1/(0.1 + 30^{0.2}) = 0.48$

Factor: $\varphi_0 = \varphi_{\mathrm{RH}}\beta(f_{cm})\beta(t_0) = 1.43 \times 2.21 \times 0.48 = 1.52$

Factor: $\beta_H = 1.5\left[1 + (0.012RH)^{18}\right]h_0 + 250\alpha_3$

$=1.5 \times \left[1 + (0.012 \times 50)^{18}\right] \times 219 + 250 \times 0.78 = 523$

Factor: $\beta_c(t, t_0) = \left(\frac{t - t_0}{\beta_H + t - t_0}\right)^{0.3} = \left(\frac{\infty - 14}{522 + \infty - 14}\right)^{0.3} = 1.0$

Creep coefficient: $\varphi_t = \varphi_0\beta_c(t, t_0) = 1.52 \times 1.0 = 1.52$

- Elastic modulus of concrete considering long-term effect

$$E_{c,eff} = E_{cm}\frac{1}{1 + (N_{G,Ed}/N_{Ed})\varphi_t} = \frac{37.3}{1 + (5000/12000) \times 1.52} = 22.8 \text{ GPa}$$

- Effective flexural stiffness of cross-section

$$(EI)_{eff,y} = E_a I_{ay} + 0.6E_{c,eff}I_{cy}$$

$$= [210 \times 193152 + 0.6 \times 22.8 \times 526848] \times 10^4 = 4.78 \times 10^{11} \text{ kN} \cdot \text{mm}^2$$

$$(EI)_{eff,z} = E_a I_{az} + 0.6E_{c,eff}I_{cz}$$

$$= [210 \times 102272 + 0.6 \times 22.8 \times 217728] \times 10^4 = 2.45 \times 10^{11} \text{ kN} \cdot \text{mm}^2$$

- Elastic critical Euler buckling resistance

$$N_{cr,y} = \frac{\pi^2 (EI)_{eff,y}}{L_{eff}^2} = \frac{\pi^2 \times 4.78 \times 10^{11}}{6000^2} = 130977 \text{ kN}$$

$$N_{cr,z} = \frac{\pi^2 (EI)_{eff,z}}{L_{eff}^2} = \frac{\pi^2 \times 2.45 \times 10^{11}}{6000^2} = 67053 \text{ kN}$$

- Characteristic plastic resistance of cross-section

$$N_{pl,Rk} = A_a f_y + A_c f_{ck} = [384 \times 355 + 2016 \times 50] \times 10^{-1} = 23712 \text{ kN}$$

- Relative slenderness ratio

$$\bar{\lambda}_y = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{23712}{130977}} = 0.425$$

$$\bar{\lambda}_z = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{23712}{67053}} = 0.595$$

- Buckling curves and buckling reduction factors

The buckling curves about both axis are "a". Thus, the imperfection factor is $\alpha = 0.21$.

$$\bar{\lambda} = \max\left(\bar{\lambda}_y, \bar{\lambda}_z\right) = \max(0.425, 0.595) = 0.595$$

$$\Phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right] = 0.5\left[1 + 0.21 \times (0.595 - 0.2) + 0.595^2\right] = 0.718$$

$$\chi = \min\left(\frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, 1.0\right) = \min\left(\frac{1}{0.718 + \sqrt{0.718^2 - 0.595^2}}, 1.0\right) = 0.892$$

- Simplified Interaction Curves

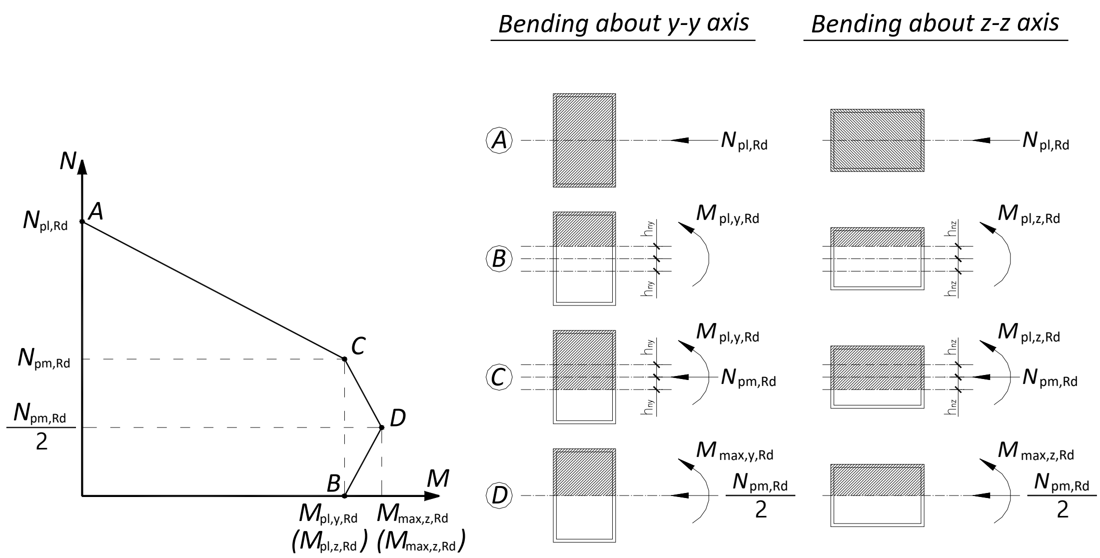


Figure 10: Simplified interaction curve for rectangular CFST columns

1) Point A (0, $N_{pl,Rd}$):

Confinement effect is not taken into account for rectangular CFST column, thus

$$N_{pl,Rd} = A_a f_{yd} + A_c f_{cd} = \left[384 \times 355 + 2016 \times 33.3\right] \times 10^{-1} = 20352 \text{ kN}$$

Buckling resistance: $\chi N_{pl,Rd} = 0.892 \times 20352 = 18154 \text{ kN}$

2) Point B ($M_{pl,y,Rd}$, 0) & ($M_{pl,z,Rd}$, 0):

The position of neutral axis are determined from

$$h_{ny} = \frac{A_c f_{cd}}{2Bf_{cd} + 4t_a\left(2f_{yd} - f_{cd}\right)} = \frac{201600 \times 33.3}{2 \times 400 \times 33.3 + 4 \times 20 \times \left(2 \times 355 - 33.3\right)} = 83.2 \text{ mm}$$

$$h_{nz} = \frac{A_c f_{cd}}{2Hf_{cd} + 4t_a\left(2f_{yd} - f_{cd}\right)} = \frac{201600 \times 33.3}{2 \times 600 \times 33.3 + 4 \times 20 \times \left(2 \times 355 - 33.3\right)} = 71.4 \text{ mm}$$

The plastic modulus of steel tube and concrete in the height of $2h_n$, bending about centreline of the cross-section are calculated as:

$$W_{ay,n} = 2t_a h_{ny}^2 = 2 \times 20 \times 83.2^2 \times 10^{-3} = 276.9 \text{ cm}^3$$

$$W_{az,n} = 2t_a h_{nz}^2 = 2 \times 20 \times 71.4^2 \times 10^{-3} = 204 \text{ cm}^3$$

$$W_{cy,n} = \left(B - 2t_a\right) h_{ny}^2 = \left(400 - 2 \times 20\right) \times 83.2^2 \times 10^{-3} = 2492 \text{ cm}^3$$

$$W_{cz,n} = \left(H - 2t_a\right) h_{nz}^2 = \left(600 - 2 \times 20\right) \times 71.4^2 \times 10^{-3} = 2855 \text{ cm}^3$$

Taking moment about the centreline of the cross-section, the plastic bending resistance is determined from:

$$\begin{aligned} M_{pl,y,Rd} &= \left(W_{ay} - W_{ay,n}\right) f_{yd} + 0.5\left(W_{cy} - W_{cy,n}\right) f_{cd} \\ &= \left[\left(7776 - 276.9\right) \times 355 + 0.5 \times \left(28224 - 2492\right) \times 33.3\right] \times 10^{-3} \\ &= 3091 \text{ kN} \cdot \text{m} \end{aligned}$$

$$M_{pl,z,Rd} = \left(W_{az} - W_{az,n}\right) f_{yd} + 0.5\left(W_{cz} - W_{cz,n}\right) f_{cd}$$

$$= \left[\left(5856 - 204\right) \times 355 + 0.5 \times \left(18144 - 2855\right) \times 33.3\right] \times 10^{-3}$$

$$= 2261 \text{ kN} \cdot \text{m}$$

3) Point C ($M_{pl,y,Rd}$, $N_{pm,Rd}$) & ($M_{pl,z,Rd}$, $N_{pm,Rd}$):

$$N_{pm,Rd} = A_c f_{cd} = 201600 \times 33.3 \times 10^{-3} = 6720 \text{ kN}$$

4) Point D ($M_{max,y,Rd}$, $N_{pm,Rd}/2$) & ($M_{max,z,Rd}$, $N_{pm,Rd}/2$):

$$M_{\max,y,Rd} = W_{ay} f_{yd} + 0.5 W_{cy} f_{cd}$$

$$= \left[7776 \times 355 + 0.5 \times 28224 \times 33.3\right] \times 10^{-3}$$

$$= 3231 \text{ kN} \cdot \text{m}$$

$$M_{\max,z,Rd} = W_{az} f_{yd} + 0.5 W_{cz} f_{cd}$$

$$= \left[5856 \times 355 + 0.5 \times 18144 \times 33.3\right] \times 10^{-3}$$

$$= 2381 \text{ kN} \cdot \text{m}$$

- Steel contribution ratio

$$\delta = A_a f_{yd} / N_{pl,Rd} = 384 \times 355 \times 10^{-1} / 20352$$

$$= 0.67 < 0.9$$

- Check for resistance of column in axial compression

$$\frac{N_{Ed}}{\chi N_{pl,Rd}} = \frac{12000}{0.892 \times 20352} = 0.661 < 1.0$$

Thus, the buckling resistance under axial compression is adequate!

- Check for resistance of column in combined compression and biaxial bending moments

The design value of effective flexural stiffness with long-term effect is calculated from:

$$(EI)_{eff,II,y} = K_0\left(E_a I_{ay} + K_{e,II} E_{c,m} I_{cy}\right) = 0.9 \times \left[210 \times 193152 + 0.5 \times 22.8 \times 526848\right] \times 10^4$$

$$= 4.19 \times 10^{11} \text{ kN} \cdot \text{mm}^2$$

$$(EI)_{eff,II,z} = K_0\left(E_a I_{az} + K_{e,II} E_{c,m} I_{cz}\right) = 0.9 \times \left[210 \times 102272 + 0.5 \times 22.8 \times 217728\right] \times 10^4$$

$$= 2.16 \times 10^{11} \text{ kN} \cdot \text{mm}^2$$

Thus, the critical normal forces with effective length taken as the system length of column are determined from:

$$N_{cr,\text{eff,y}} = \frac{\pi^2 (EI)_{eff,\text{II,y}}}{L^2} = \frac{\pi^2 \times 4.19\times 10^{11}}{6000^2} = 114913 \text{ kN}$$

$$N_{cr,\text{eff,z}} = \frac{\pi^2 (EI)_{eff,\text{II,z}}}{L^2} = \frac{\pi^2 \times 2.16\times 10^{11}}{6000^2} = 59122 \text{ kN}$$

The second-order effect should be considered for both moments from first-order analysis and moment from imperfection, as shown in Figure 6. According to the buckling curve "a" and refer to Table 3.3 of BC4, the initial imperfections about y-y axis and z-z axis are:

$$e_{0,y} = L/300 = 6000/300 = 20 \text{ mm}$$

$$e_{0,z} = L/300 = 6000/300 = 20 \text{ mm}$$

Accordingly, the bending moments by the initial imperfections are determined as:

$$M_{0,y} = N_{Ed}e_{0,y} = 12000\times 20/1000 = 240 \text{ kN}\cdot\text{m}$$

$$M_{0,z} = N_{Ed}e_{0,z} = 12000\times 20/1000 = 240 \text{ kN}\cdot\text{m}$$

According to the moment diagram by the initial imperfection, the factor β_0 for determination of moment to second-order effect is equal to 1.0. Thus, the amplification factors for the moments by the imperfection are calculated from:

$$k_{0,y} = \frac{\beta_0}{1 - N_{Ed}/N_{cr,\text{eff,y}}} = \frac{1.0}{1-12000/114913} = 1.117$$

$$k_{0,z} = \frac{\beta_0}{1 - N_{Ed}/N_{cr,\text{eff,z}}} = \frac{1.0}{1-12000/59122} = 1.255$$

According to the first-order design moment diagram, the ratio of end moments is calculated as:

$$r_y = M_{t,y}/M_{b,y} = -550/900 = -0.611$$

$$r_z = M_{t,z}/M_{b,z} = 350/(-600) = -0.583$$

Thus, the factors for determination of moment to second-order effect are determined:

$$\beta_{1,y} = \max\left(0.66 + 0.44 r_y, 0.44\right) = \max\left(0.66 + 0.44 \times \left(-0.611\right), 0.44\right) = 0.44$$

$$\beta_{1,z} = \max\left(0.66 + 0.44 r_z, 0.44\right) = \max\left(0.66 + 0.44 \times \left(-0.583\right), 0.44\right) = 0.44$$

Thus, the amplification factors for the moment by the imperfection are calculated from:

$$k_{1,y} = \frac{\beta_{1,y}}{1 - N_{Ed} / N_{cr,\mathrm{eff,y}}} = \frac{0.44}{1 - 12000/114913} = 0.491$$

$$k_{1,z} = \frac{\beta_{1,z}}{1 - N_{Ed} / N_{cr,\mathrm{eff,z}}} = \frac{0.44}{1 - 12000/59122} = 0.552$$

Thus, the design moments, considering second-order effect, are calculated as:

$$\begin{aligned} M_{y,Ed} &= Max\left[k_{0,y} M_{0,y} + k_{1,y} Max\left(\left|M_{t,y}\right|, \left|M_{b,y}\right|\right), Max\left(\left|M_{t,y}\right|, \left|M_{b,y}\right|\right)\right] \\ &= Max\left[1.117 \times 240 + 0.491 \times Max\left(\left|-550\right|, \left|900\right|\right), Max\left(\left|-550\right|, \left|900\right|\right)\right] = 900 \ \mathrm{kN \cdot m} \end{aligned}$$

$$\begin{aligned} M_{z,Ed} &= Max\left[k_{0,z} M_{0,z} + k_{1,z} Max\left(\left|M_{t,z}\right|, \left|M_{b,z}\right|\right), Max\left(\left|M_{t,z}\right|, \left|M_{b,z}\right|\right)\right] \\ &= Max\left[1.255 \times 240 + 0.552 \times Max\left(\left|350\right|, \left|-600\right|\right), Max\left(\left|350\right|, \left|-600\right|\right)\right] = 632 \ \mathrm{kN \cdot m} \end{aligned}$$

For $N_{Ed} = 12000\ kN > N_{pm,Rd} = 6720\ kN$, the values for determining the plastic bending resistances $M_{pl,N,y,Rd}$ and $M_{pl,N,z,Rd}$ taking into account the normal force N_{Ed} are calculated from:

$$\mu_{dy} = \mu_{dz} = \frac{N_{pl,Rd} - N_{Ed}}{N_{pl,Rd} - N_{pm,Rd}} = \frac{20352 - 12000}{20352 - 6720} = 0.613$$

$$\frac{M_{y,Ed}}{M_{pl,N,y,Rd}} = \frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} = \frac{900}{0.613 \times 3091} = 0.475 < \alpha_{M,y} = 0.9$$

$$\frac{M_{z,Ed}}{M_{pl,N,z,Rd}} = \frac{M_{z,Ed}}{\mu_{dz} M_{pl,z,Rd}} = \frac{632}{0.613 \times 2261} = 0.456 < \alpha_{M,z} = 0.9$$

$$\frac{M_{y,Ed}}{\mu_{dy} M_{pl,y,Rd}} + \frac{M_{z,Ed}}{\mu_{dz} M_{pl,z,Rd}} = \frac{900}{0.613 \times 3091} + \frac{632}{0.613 \times 2261} = 0.932 < 1.0$$

Thus, the resistance for combined axial compression and biaxial bending is adequate. The external design force and bending moment, and M-N interaction curves are plotted in Figure 11, Figure 12 and Figure 13.

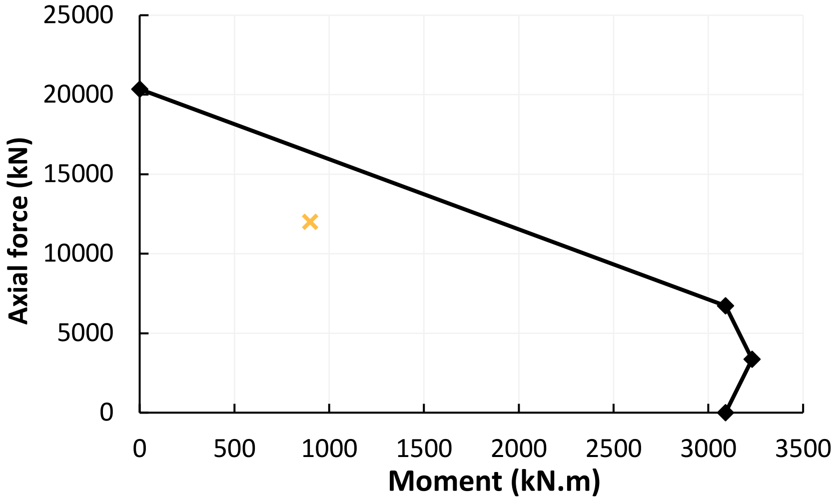


Figure 11: Design M-N curve for bending about y-y axis

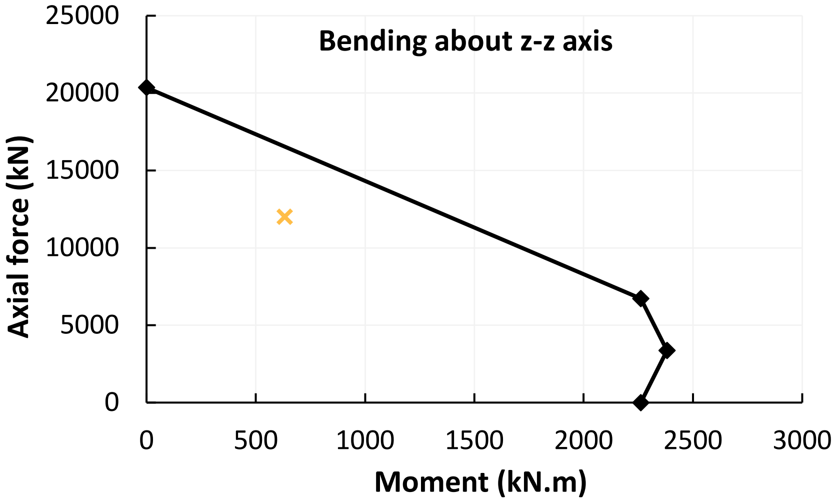


Figure 12: Design M-N curve for bending about z-z axis

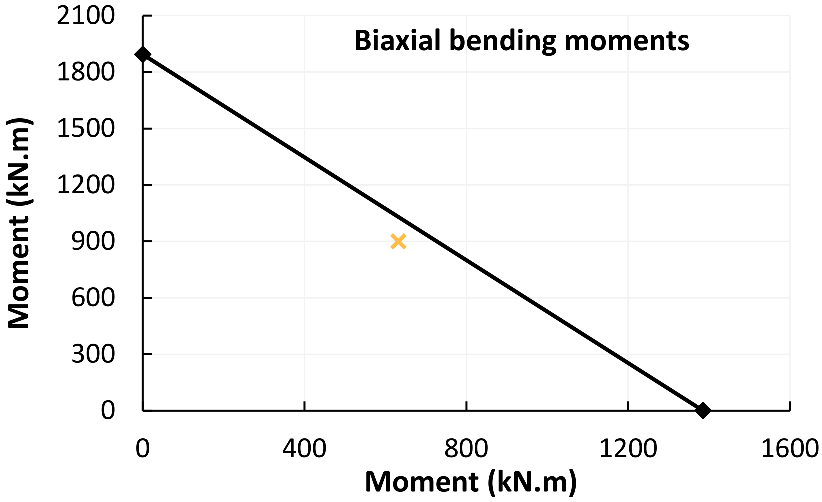


Figure 13: Check for bi-axial bending

3.3 RHS 400x600x20 - S355 steel tube infilled with C90/105 concrete

In this section, the high strength concrete C50/60 is replaced by higher strength concrete C90/105. The steel grade is not changed.

- Design strength

Effective compressive strength and modulus of elasticity are taken from to Table 2.2, Eq.(2.2) and Eq.(2.3) of BC4.

$$f_{ck} = 72 \text{ N/mm}^2; \quad E_{cm} = 41.1 \text{ GPa}$$

$$f_{cd} = f_{ck}/\gamma_c = 72/1.5 = 48 \text{ N/mm}^2; \; f_{cm} = f_{ck} + 8 = 72 + 8 = 80 \text{ N/mm}^2$$

- Elastic modulus of concrete considering long-term effect

Creep coefficient could be similarly determined as $\varphi_t = 1.12$

$$E_{c,eff} = \frac{E_{cm}}{1+\left(N_{G,Ed}/N_{Ed}\right)\varphi_t} = \frac{41.1}{1+\left(5000/12000\right)\times 1.12} = 28 \text{ GPa}$$

- Effective flexural stiffness of cross-section

$$(EI)_{eff,y} = E_a I_{ay} + 0.6 E_{c,eff} I_{cy} = \left[210\times 193152 + 0.6\times 28\times 526848\right]\times 10^4 = 4.94\times 10^{11} \text{ kN}\cdot\text{mm}^2$$

$$(EI)_{eff,z} = E_a I_{az} + 0.6 E_{c,eff} I_{cz} = \left[210\times 102272 + 0.6\times 28\times 217728\right]\times 10^4 = 2.51\times 10^{11} \text{ kN}\cdot\text{mm}^2$$

- Characteristic plastic resistance of cross-section

$$N_{cr,y} = \frac{\pi^2 (EI)_{eff,y}}{L_{eff}^2} = \frac{\pi^2\times 4.94\times 10^{11}}{6000^2} = 135431 \text{ kN}$$

$$N_{cr,z} = \frac{\pi^2 (EI)_{eff,z}}{L_{eff}^2} = \frac{\pi^2\times 2.51\times 10^{11}}{6000^2} = 68893 \text{ kN}$$

$$N_{pl,Rk} = A_a f_y + A_c f_{ck} = \left[384\times 355 + 2016\times 90\right]\times 10^{-1} = 28147 \text{ kN}$$

- Relative slenderness ratio, buckling curves and buckling reduction factors

$$\overline{\lambda}_y = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{28147}{135431}} = 0.456; \; \overline{\lambda}_z = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{28147}{68893}} = 0.639$$

$$\bar{\lambda} = \max\left(\bar{\lambda}_y, \bar{\lambda}_z\right) = 0.639$$

$$\Phi = 0.5\left[1+\alpha\left(\bar{\lambda}-0.2\right)+\bar{\lambda}^2\right] = 0.5\left[1+0.21\times\left(0.639-0.2\right)+0.639^2\right] = 0.75$$

$$\chi = \min\left(\frac{1}{\Phi+\sqrt{\Phi^2-\bar{\lambda}^2}},1.0\right) = \min\left(\frac{1}{0.75+\sqrt{0.75^2-0.639^2}},1.0\right) = 0.875$$

$$N_{pl,Rd} = A_a f_{yd} + A_c f_{cd} = \left[384\times355+2016\times41.1\right]\times10^{-1} = 23309 \text{ kN}$$

Buckling resistance: $\chi N_{pl,Rd} = 0.875\times23309 = 20395 \text{ kN}$

- Steel contribution ratio

$$\delta = A_a f_{yd} / N_{pl,Rd} = 384\times355\times10^{-1}/23309 = 0.585 < 0.9$$

- M-N interaction curve

$$h_{ny} = \frac{A_c f_{cd}}{2Bf_{cd}+4t_a\left(2f_{yd}-f_{cd}\right)} = \frac{201600\times41.1}{2\times400\times41.1+4\times20\times\left(2\times355-41.1\right)} = 105.9 \text{ mm}$$

$$h_{nz} = \frac{A_c f_{cd}}{2Hf_{cd}+4t_a\left(2f_{yd}-f_{cd}\right)} = \frac{201600\times41.1}{2\times600\times41.1+4\times20\times\left(2\times355-41.1\right)} = 87.5 \text{ mm}$$

$$W_{ay,n} = 2t_a h_{ny}^2 = 2\times20\times105.9^2\times10^{-3} = 448.6 \text{ cm}^3$$

$$W_{az,n} = 2t_a h_{nz}^2 = 2\times20\times87.5^2\times10^{-3} = 306.3 \text{ cm}^3$$

$$W_{cy,n} = \left(B-2t_a\right)h_{ny}^2 = \left(400-2\times20\right)\times105.9^2\times10^{-3} = 4037.3 \text{ cm}^3$$

$$W_{cz,n} = \left(H-2t_a\right)h_{nz}^2 = \left(600-2\times20\right)\times87.5^2\times10^{-3} = 4287.5 \text{ cm}^3$$

$$\begin{aligned} M_{pl,y,Rd} &= \left(W_{ay}-W_{ay,\text{n}}\right)f_{yd}+0.5\left(W_{cy}-W_{\text{cy,n}}\right)f_{cd} \\ &= \left[\left(7776-448.6\right)\times355+0.5\times\left(28224-4037.3\right)\times41.1\right]\times10^{-3} = 3182 \text{ kN}\cdot\text{m} \end{aligned}$$

$$\begin{aligned} M_{pl,z,Rd} &= \left(W_{az}-W_{az,\text{n}}\right)f_{yd}+0.5\left(W_{cz}-W_{\text{cz,n}}\right)f_{cd} \\ &= \left[\left(5856-306.3\right)\times355+0.5\times\left(18144-4287.5\right)\times41.1\right]\times10^{-3} = 2303 \text{ kN}\cdot\text{m} \end{aligned}$$

$$N_{pm,Rd} = A_c f_{cd} = 201600\times41.1\times10^{-3} = 9677 \text{ kN}$$

$$M_{\max,y,Rd} = W_{ay} f_{yd} + 0.5 W_{cy} f_{cd}$$
$$= \left[7776 \times 355 + 0.5 \times 28224 \times 41.1\right] \times 10^{-3} = 3438 \text{ kN} \cdot \text{m}$$

$$M_{\max,z,Rd} = W_{az} f_{yd} + 0.5 W_{cz} f_{cd}$$
$$= \left[5856 \times 355 + 0.5 \times 18144 \times 41.1\right] \times 10^{-3} = 2514 \text{ kN} \cdot \text{m}$$

3.4 RHS 400x600x20 - S500 steel tube infilled with C50/60 concrete

In this section, the mild steel S355 is replaced by the high tensile steel (HTS) S500, the concrete grade is not changed.

- Plastic resistance of cross-section

$$N_{pl,Rk} = A_a f_y + A_c f_{ck} = \left[384 \times 500 + 2016 \times 50\right] \times 10^{-1} = 29280 \text{ kN}$$

$$\overline{\lambda}_y = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{29280}{130977}} = 0.473\,;\ \overline{\lambda}_z = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{29280}{67053}} = 0.661$$

$$\overline{\lambda} = \max\left(\overline{\lambda}_y, \overline{\lambda}_z\right) = 0.661$$

$$\Phi = 0.5\left[1 + \alpha\left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right] = 0.5\left[1 + 0.21 \times \left(0.661 - 0.2\right) + 0.661^2\right] = 0.767$$

$$\chi = \min\left(\frac{1}{\Phi + \sqrt{\Phi^2 - \overline{\lambda}^2}}, 1.0\right) = \min\left(\frac{1}{0.767 + \sqrt{0.767^2 - 0.661^2}}, 1.0\right) = 0.865$$

$$N_{pl,Rd} = A_a f_{yd} + A_c f_{cd} = \left[384 \times 500 + 2016 \times 33.3\right] \times 10^{-1} = 25920 \text{ kN}$$

Buckling resistance: $\chi N_{pl,Rd} = 0.865 \times 25920 = 22421 \text{ kN}$

- Steel contribution ratio

$$\delta = A_a f_{yd} / N_{pl,Rd} = 384 \times 500 \times 10^{-1} / 22421 = 0.741 < 0.9$$

- M-N interaction curve

$$h_{ny} = \frac{A_c f_{cd}}{2B f_{cd} + 4 t_a \left(2 f_{yd} - f_{cd}\right)} = \frac{201600 \times 33.3}{2 \times 400 \times 33.3 + 4 \times 20 \times \left(2 \times 500 - 33.3\right)} = 64.6 \text{ mm}$$

$$h_{nz} = \frac{A_c f_{cd}}{2H f_{cd} + 4 t_a \left(2 f_{yd} - f_{cd}\right)} = \frac{201600 \times 33.3}{2 \times 600 \times 33.3 + 4 \times 20 \times \left(2 \times 500 - 33.3\right)} = 57.3 \text{ mm}$$

$$W_{ay,n} = 2t_a h_{ny}^2 = 2 \times 20 \times 64.6^2 \times 10^{-3} = 170 \text{ cm}^3$$

$$W_{az,n} = 2t_a h_{nz}^2 = 2 \times 20 \times 57.3^2 \times 10^{-3} = 131.3 \text{ cm}^3$$

$$W_{cy,n} = \left(B - 2t_a\right) h_{ny}^2 = \left(400 - 2 \times 20\right) \times 64.6^2 \times 10^{-3} = 1502.3 \text{ cm}^3$$

$$W_{cz,n} = \left(H - 2t_a\right) h_{nz}^2 = \left(600 - 2 \times 20\right) \times 57.3^2 \times 10^{-3} = 1838.6 \text{ cm}^3$$

$$M_{pl,\text{y},Rd} = \left(W_{ay} - W_{ay,\text{n}}\right) f_{yd} + 0.5\left(W_{cy} - W_{\text{cy,n}}\right) f_{cd}$$
$$= \left[\left(7776 - 170\right) \times 500 + 0.5 \times \left(28224 - 1502.3\right) \times 33.3\right] \times 10^{-3} = 4250 \text{ kN} \cdot \text{m}$$

$$M_{pl,\text{z},Rd} = \left(W_{az} - W_{az,\text{n}}\right) f_{yd} + 0.5\left(W_{cz} - W_{\text{cz,n}}\right) f_{cd}$$
$$= \left[\left(5856 - 131.3\right) \times 500 + 0.5 \times \left(18144 - 1838.6\right) \times 33.3\right] \times 10^{-3} = 3134 \text{ kN} \cdot \text{m}$$

$$N_{pm,Rd} = A_c f_{cd} = 201600 \times 33.3 \times 10^{-3} = 6720 \text{ kN}$$

$$M_{\max,\text{y},Rd} = W_{ay} f_{yd} + 0.5 W_{cy} f_{cd}$$
$$= \left[7776 \times 500 + 0.5 \times 28224 \times 33.3\right] \times 10^{-3} = 4358 \text{ kN} \cdot \text{m}$$

$$M_{\max,z,Rd} = W_{az} f_{yd} + 0.5 W_{cz} f_{cd}$$
$$= \left[5856 \times 500 + 0.5 \times 18144 \times 33.3\right] \times 10^{-3} = 3230 \text{ kN} \cdot \text{m}$$

3.5 RHS 400x600x20 - S500 steel tube infilled with C90/105 concrete

In this section, the mild steel S355 is replaced by the high tensile steel (HTS) S500, and the normal strength concrete C50/60 is replaced by high strength concrete C90/105.

- Plastic resistance of cross-section

$$N_{pl,Rk} = A_a f_y + A_c f_{ck} = \left[384 \times 500 + 2016 \times 90\right] \times 10^{-1} = 33715 \text{ kN}$$

$$\overline{\lambda}_y = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{33715}{135431}} = 0.499\,;\ \overline{\lambda}_z = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{33715}{68893}} = 0.7$$

$$\overline{\lambda} = \max\left(\overline{\lambda}_y, \overline{\lambda}_z\right) = 0.7$$

$$\Phi = 0.5\left[1 + \alpha\left(\overline{\lambda} - 0.2\right) + \overline{\lambda}^2\right] = 0.5\left[1 + 0.21 \times \left(0.7 - 0.2\right) + 0.7^2\right] = 0.797$$

$$\chi = \min\left(\frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, 1.0\right) = \min\left(\frac{1}{0.797 + \sqrt{0.797^2 - 0.7^2}}, 1.0\right) = 0.848$$

$$N_{pl,Rd} = A_a f_{yd} + A_c f_{cd} = \left[384 \times 500 + 2016 \times 41.1\right] \times 10^{-1} = 28877 \text{ kN}$$

Buckling resistance: $\chi N_{pl,Rd} = 0.848 \times 28877 = 24488 \text{ kN}$

- Steel contribution ratio

$$\delta = A_a f_{yd} / N_{pl,Rd} = 384 \times 500 \times 10^{-1} / 24488 = 0.665 < 0.9$$

- M-N interaction curve

$$h_{ny} = \frac{A_c f_{cd}}{2Bf_{cd} + 4t_a\left(2f_{yd} - f_{cd}\right)} = \frac{201600 \times 41.1}{2 \times 400 \times 41.1 + 4 \times 20 \times \left(2 \times 500 - 41.1\right)} = 84.5 \text{ mm}$$

$$h_{nz} = \frac{A_c f_{cd}}{2Hf_{cd} + 4t_a\left(2f_{yd} - f_{cd}\right)} = \frac{201600 \times 41.1}{2 \times 600 \times 41.1 + 4 \times 20 \times \left(2 \times 500 - 41.1\right)} = 72.3 \text{ mm}$$

$$W_{ay,n} = 2t_a h_{ny}^2 = 2 \times 20 \times 84.5^2 \times 10^{-3} = 285.6 \text{ cm}^3$$

$$W_{az,n} = 2t_a h_{nz}^2 = 2 \times 20 \times 72.3^2 \times 10^{-3} = 209.1 \text{ cm}^3$$

$$W_{cy,n} = \left(B - 2t_a\right) h_{ny}^2 = \left(400 - 2 \times 20\right) \times 84.5^2 \times 10^{-3} = 2570.5 \text{ cm}^3$$

$$W_{cz,n} = \left(H - 2t_a\right) h_{nz}^2 = \left(600 - 2 \times 20\right) \times 72.3^2 \times 10^{-3} = 2927.3 \text{ cm}^3$$

$$\begin{aligned} M_{pl,y,Rd} &= \left(W_{ay} - W_{ay,\mathrm{n}}\right) f_{yd} + 0.5\left(W_{cy} - W_{\mathrm{cy,n}}\right) f_{cd} \\ &= \left[\left(7776 - 285.6\right) \times 500 + 0.5 \times \left(28224 - 2570.5\right) \times 41.1\right] \times 10^{-3} = 4361 \text{ kN} \cdot \text{m} \end{aligned}$$

$$\begin{aligned} M_{pl,z,Rd} &= \left(W_{az} - W_{az,\mathrm{n}}\right) f_{yd} + 0.5\left(W_{cz} - W_{\mathrm{cz,n}}\right) f_{cd} \\ &= \left[\left(5856 - 209.1\right) \times 500 + 0.5 \times \left(18144 - 2927.3\right) \times 41.1\right] \times 10^{-3} = 3188 \text{ kN} \cdot \text{m} \end{aligned}$$

$$N_{pm,Rd} = A_c f_{cd} = 201600 \times 41.1 \times 10^{-3} = 9677 \text{ kN}$$

$$\begin{aligned} M_{\mathrm{max,y},Rd} &= W_{ay} f_{yd} + 0.5 W_{cy} f_{cd} \\ &= \left[7776 \times 500 + 0.5 \times 28224 \times 41.1\right] \times 10^{-3} = 4565 \text{ kN} \cdot \text{m} \end{aligned}$$

$$M_{\max,z,Rd} = W_{az} f_{yd} + 0.5 W_{cz} f_{cd}$$

$$= \left[5856 \times 500 + 0.5 \times 18144 \times 41.1\right] \times 10^{-3} = 3363 \text{ kN} \cdot \text{m}$$

3.6 Comparison and summary

The design resistances are compared for the aforementioned four rectangular composite sections as shown in Table 2, Figure 14 and Figure 15. The composite section with steel tube of S355 and concrete of C50/60 is referred to for comparison.

By using high strength concrete C90/105 replacing normal strength concrete C50/60, the axial buckling resistance ($\chi N_{pl,Rd}$) of the CFST column is improved by 12.3%, and the increase of moment resistances ($M_{pl,y,Rd}$, $M_{pl,z,Rd}$, $M_{max,y,Rd}$, $M_{max,z,Rd}$) are smaller (less than 7%).

Table 2: Comparison between rectangular CFST columns with various material strengths

Sections	Steel contribution ratios	Design resistances				
		$\chi N_{pl,Rd}$	$M_{pl,y,Rd}$	$M_{pl,z,Rd}$	$M_{max,y,Rd}$	$M_{max,z,Rd}$
S355+C50/60	0.670	0	0	0	0	0
S355+C90/105	0.585	12.3%	2.9%	1.8%	6.4%	5.6%
S500+C50/60	0.741	23.5%	37.5%	38.6%	34.9%	35.6%
S500+C90/105	0.665	34.9%	41.1%	41.0%	41.3%	41.2%

By using high strength steel S500 to replace S355 steel, the axial buckling resistance is improved by 23.5%, and the increase of moment resistances are larger (higher than 34.9%). By using high strength concrete C90/105 to replace normal strength concrete C50/60 and use of S500 steel to replace S355 steel, the axial buckling resistance is further improved to 34.9%, and the increase of moment resistances are more than 40%.

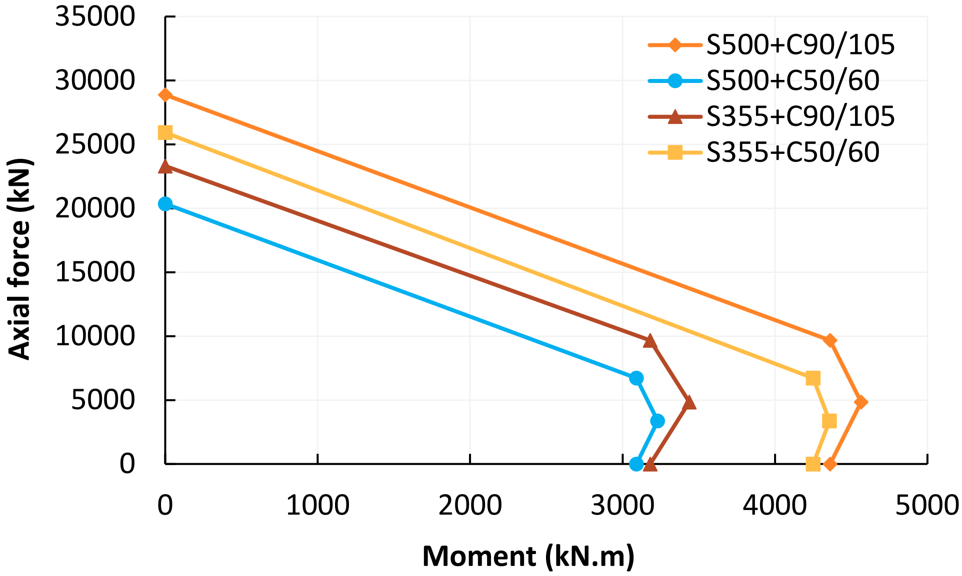


Figure 14: Design M-N interaction curves about y-y axis

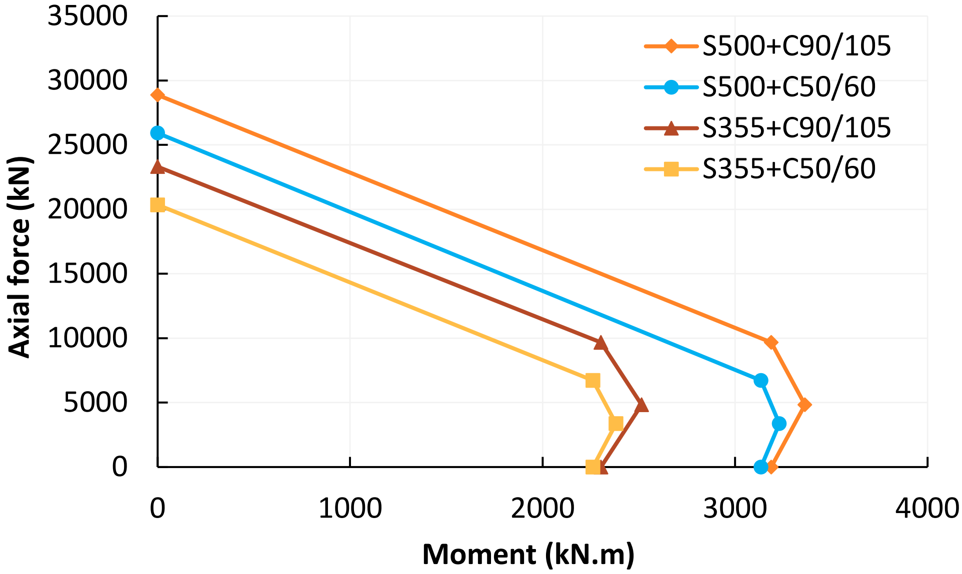


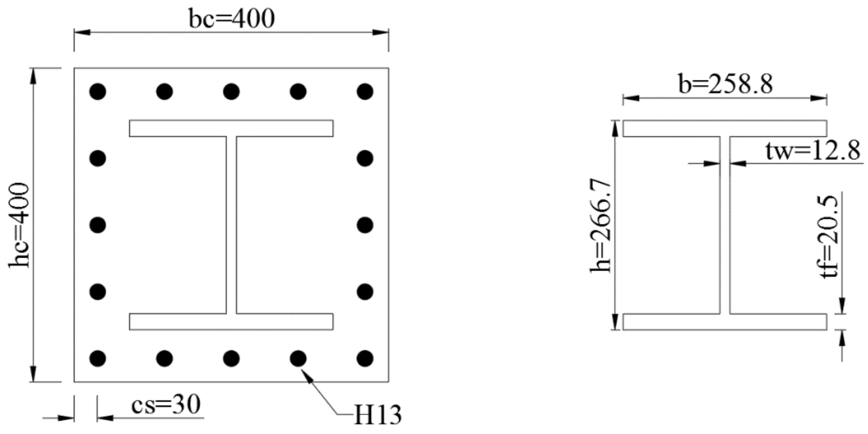
Figure 15: Design M-N interaction curves about z-z axis

4 Example 4

4.1 General

Concrete encased steel member subject to axial compression and bending about the major axis. The following steel, concrete and reinforcing materials are used:

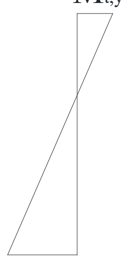
a) UC 254x254x107 S355 sections with C50/60 concrete and G5000 reinforcements
b) UC 254x254x107 S355 sections with C90/105 concrete and G500 reinforcements
c) UC 254x254x107 S500 sections with C50/60 concrete and G500 reinforcements



4.2 UC 254x254x107 S355 sections with C50/60 concrete and G500 reinforcements

- Design parameters

Concrete	C50/60, f_{ck}=50 N/mm^2
Embedded steel section	Grade S355, f_{ek}=355 N/mm^2
Reinforcements	Grade 500, f_{sk}=500 N/mm^2
Column system length	L=4000 mm
Effective length	L_{eff} =4000 mm
Total design axial load	N_{Ed}=6000 kN
Design axial load that is permanent	$N_{G,Ed}$=4000 kN



Design moment at bottom around y-y axis $M_{b,y}$=400 kN.m

Design moment at top around y-y axis $M_{t,y}$=-200 kN.m

- Design strength and modulus

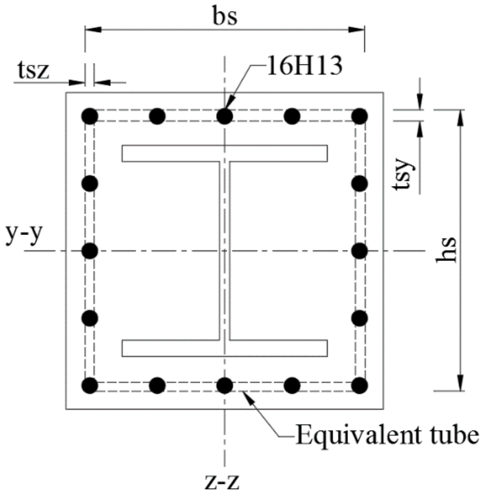
$$f_{sd} = f_{sk}/\gamma_s = 500/1.15 = 435\,\mathrm{N/mm^2}$$
$$f_{ed} = f_{ek}/\gamma_a = 355/1.0 = 355\,\mathrm{N/mm^2}$$
$$f_{cd} = f_{ck}/\gamma_c = 50/1.5 = 33.3\,\mathrm{N/mm^2}$$
$$f_{cm} = f_{ck} + 8 = 50 + 8 = 58\,\mathrm{N/mm^2}$$
$$E_s = E_e = 210\,\mathrm{GPa}$$
$$E_{cm} = 22(f_{cm}/10)^{0.3} = 22(58/10)^{0.3} = 37.3\,\mathrm{GPa}$$

- Cross sectional areas

$$A = b_c h_c = 400 \times 400 = 160000\,\mathrm{mm^2}$$
$$A_e = bh - (b - t_w)(h - 2t_f)$$
$$= 258.8 \times 266.7 - (258.8 - 12.8)(266.7 - 2 \times 20.5) = 13500\,\mathrm{mm^2}$$
$$A_s = 16(\pi/4)d^2 = 16 \times (\pi/4) \times 13^2 = 2120\,\mathrm{mm^2}$$
$$A_c = A - A_e - A_s = 160000 - 13500 - 2120 = 144380\,\mathrm{mm^2}$$

- Second moment of areas

For simplicity, the reinforcements are equivalently converted to a square tube based on the same cross-sectional area and position of centreline.



$$t_{sy} = 5(\pi/4)d^2/(b_c - 2cs) = 5(\pi/4)\times 13^2/(400 - 2\times 30) = 2 \text{ mm}$$

$$t_{sz} = 5(\pi/4)d^2/(h_c - 2cs) = 5(\pi/4)\times 13^2/(400 - 2\times 30) = 2 \text{ mm}$$

$$h_s = (h_c - 2cs) + t_{sy} = 342 \text{ mm}$$

$$b_s = (b_c - 2cs) + t_{sz} = 342 \text{ mm}$$

$$I_{sy} = \frac{1}{12}\left[b_s h_s^3 - (b_s - 2t_{sz})(h_s - 2t_{sy})^3\right] = 5115 \text{ cm}^4$$

$$I_{sz} = \frac{1}{12}\left[h_s b_s^3 - (h_s - 2t_{sy})(b_s - 2t_{sz})^3\right] = 5115 \text{ cm}^4$$

$$I_{ey} = \frac{1}{12}\left[bh^3 - (b - t_w)(h - 2t_f)^3\right] = \frac{1}{12}\left[258.8\times 266.7^3 - (258.8 - 12.8)(266.7 - 2\times 20.5)^3\right] = 17343 \text{ cm}^4$$

$$I_{ez} = \frac{1}{12}\left[2t_f b^3 + (h - 2t_f)t_w^3\right] = \frac{1}{12}\left[2\times 20.5\times 258.8^3 + (266.7 - 2\times 20.5)\times 12.8^3\right] = 5926 \text{ cm}^4$$

$$I_{cy} = I - I_{sy} - I_{ey} = 213333 - 5115 - 17343 = 190876 \text{ cm}^4$$

$$I_{cz} = I - I_{sz} - I_{ez} = 213333 - 5115 - 5926 = 202292 \text{ cm}^4$$

- Plastic modulus

$$W_y = b_c h_c^2 / 4 = 400^3 / 4 = 16000 \text{ cm}^3$$

$$W_z = h_c b_c^2 / 4 = 400^3 / 4 = 16000 \text{ cm}^3$$

$$W_{sy} = \frac{1}{4}\left[b_s h_s^2 - (b_s - 2t_{sz})(h_s - 2t_{sy})^2\right] = \frac{1}{4}\left[342^3 - (342 - 2\times 2)^3\right] = 338 \text{ cm}^3$$

$$W_{sz} = \frac{1}{4}\left[h_s b_s^2 - (h_s - 2t_{sy})(b_s - 2t_{sz})^2\right] = \frac{1}{4}\left[342^3 - (342 - 2\times 2)^3\right] = 338 \text{ cm}^3$$

$$W_{ey} = \frac{1}{4}\left[bh^2 - (b - t_w)(h - 2t_f)^2\right] = \frac{1}{4}\left[258.8\times 266.7^2 - (258.8 - 12.8)(266.7 - 2\times 20.5)^2\right] = 1469 \text{ cm}^3$$

$$W_{ez} = \frac{1}{4}\left[2t_f b^2 + (h - 2t_f)t_w^2\right] = \frac{1}{4}\left[2\times 20.5\times 258.8^2 + (266.7 - 2\times 20.5)\times 12.8^2\right] = 696 \text{ cm}^3$$

$$W_{cy} = W_y - W_{sy} - W_{ey} = 16000 - 338 - 1469 = 14192 \text{ cm}^3$$

$$W_{cz} = W_z - W_{sz} - W_{ez} = 16000 - 338 - 696 = 14966 \text{ cm}^3$$

- Long-term effect

Age of concrete at loading in day: $t_0 = 30$

Age of concrete at the moment considered in days: $t = \infty$

Relative humidity of ambient environment: RH=80%

Perimeter of concrete section: $u = 2b_c + 2h_c = 1600 \text{ mm}$

Notional size of concrete section: $h_0 = 2A_c / u = 2\times 144400/1600 = 180 \text{ mm}$

Coefficient: $\alpha_1 = (35/f_{cm})^{0.7} = (35/58)^{0.7} = 0.70$

Coefficient: $\alpha_2 = (35/f_{cm})^{0.2} = (35/58)^{0.2} = 0.90$

Coefficient: $\alpha_3 = (35/f_{cm})^{0.5} = (35/58)^{0.5} = 0.78$

Factor: $\varphi_{RH} = \left(1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}}\alpha_1\right)\alpha_2 = \left(1 + \frac{1 - 80/100}{0.1\sqrt[3]{180.5}}0.70\right)\times 0.90 = 1.13$

Factor: $\beta(f_{cm}) = 16.8/\sqrt{f_{cm}} = 16.8/\sqrt{58} = 2.21$

Factor: $\beta(t_0) = 1/(0.1 + t_0^{0.2}) = 1/(0.1 + 30^{0.2}) = 0.48$

Factor: $\varphi_0 = \varphi_{RH}\beta(f_{cm})\beta(t_0) = 1.13\times 2.21\times 0.48 = 1.20$

Factor:
$$\begin{aligned}\beta_H &= 1.5\left[1+(0.012RH)^{18}\right]h_0 + 250\alpha_3 \\ &= 1.5\left[1+(0.012\times 80)^{18}\right]\times 180 + 250\times 0.78 = 595\end{aligned}$$

Factor: $\beta_c(t,t_0) = \left(\dfrac{t-t_0}{\beta_H + t - t_0}\right)^{0.3} = 1$

Creep coefficient: $\varphi_t = \varphi_0\beta_c(t,t_0) = 1.20\times 1 = 1.20$

- Elastic modulus of concrete considering long-term effect

(2) $E_{c,eff} = E_{cm}\dfrac{1}{1+(N_{G,Ed}/N_{Ed})\varphi_t} = \dfrac{37.3}{1+(4000/6000)\times 1.20} = 20.7\ \text{GPa}$

- Effective flexural stiffness of cross-section

$$\begin{aligned}(EI)_{eff,y} &= E_s I_{sy} + E_e I_{ey} + 0.6E_{c,eff} I_{cy} \\ &= \left[210\times(5115+17343)+0.6\times 20.7\times 190876\right]\times 10^4 \\ &= 7.09\times 10^{10}\ \text{kN}\cdot\text{mm}^2\end{aligned}$$

$$\begin{aligned}(EI)_{eff,z} &= E_s I_{sz} + E_e I_{ez} + 0.6E_{c,eff} I_{cz} \\ &= \left[210\times(5115+5926)+0.6\times 20.7\times 202292\right]\times 10^4 \\ &= 4.83\times 10^{10}\ \text{kN}\cdot\text{mm}^2\end{aligned}$$

- Elastic critical Euler buckling resistance

$$N_{cr,y} = \frac{\pi^2 (EI)_{eff,y}}{L_{eff}^2} = \frac{\pi^2\times 7.09\times 10^{10}}{4000^2} = 43721\ \text{kN}$$

$$N_{cr,z} = \frac{\pi^2 (EI)_{eff,z}}{L_{eff}^2} = \frac{\pi^2\times 4.83\times 10^{10}}{4000^2} = 29807\ \text{kN}$$

- Characteristic plastic resistance of cross-section

$$\begin{aligned}N_{pl,Rk} &= A_s f_{sk} + A_e f_{ek} + 0.85A_c f_{ck} \\ &= 2120\times 500 + 13500\times 355 + 0.85\times 144400\times 50 = 11990\ \text{kN}\end{aligned}$$

- Relative slenderness ratio

$$\overline{\lambda_y} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{11990}{43721}} = 0.522$$

$$\overline{\lambda_z} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{11990}{29807}} = 0.632$$

- Buckling curves and buckling reduction factors

For a fully encased steel column, the buckling curve about major axis is "b", and about minor axis is "c". Thus, the imperfection factor is $\alpha = 0.34$ and $\alpha = 0.49$, respectively.

$$\Phi_y = 0.5\left[1 + \alpha_y(\overline{\lambda_y} - 0.2) + \overline{\lambda_y}^2\right] = 0.5\left[1 + 0.34 \times (0.522 - 0.2) + 0.522^2\right] = 0.691$$

$$\chi_y = \min\left(\frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \overline{\lambda_y^2}}}, 1.0\right) = \min\left(\frac{1}{0.691 + \sqrt{0.691^2 - 0.522^2}}, 1.0\right) = 0.874$$

$$\Phi_z = 0.5\left[1 + \alpha_z(\overline{\lambda_z} - 0.2) + \overline{\lambda_z}^2\right] = 0.5\left[1 + 0.49 \times (0.632 - 0.2) + 0.632^2\right] = 0.806$$

$$\chi_z = \min\left(\frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \overline{\lambda_z^2}}}, 1.0\right) = \min\left(\frac{1}{0.806 + \sqrt{0.806^2 - 0.632^2}}, 1.0\right) = 0.766$$

$$\chi = \min(\chi_y, \chi_z) = \min(0.874, 0.766) = 0.766$$

- Simplified Interaction Curve

1) Point A (0, $N_{pl,Rd}$):

Full cross-section is under uniform compression. No bending moment is resultant from the compressive stresses on the cross-section.

$$N_{pl,Rd} = A_s f_{sd} + A_e f_{ed} + 0.85 A_c f_{cd}$$

$$= 2120 \times 435 + 13500 \times 355 + 0.85 \times 144400 \times 33.3 = 9806 \text{ kN}$$

2) Point B ($M_{pl,Rd}$, 0):

The cross-section is under partial compression and no axial force is formed. Assuming the neutral axis lies in the web of encased section ($h_n \leq h/2 - t_f$), the height of neutral axis is calculated where the areas of the equivalent tube for reinforcements, and concrete in the height of $2h_n$ are approximated as rectangles, and based on the force equilibrium between the tensile capacity of steel sections within the height of $2h_n$ is equal to the compression resistance of concrete in the compression zone. Unless other stated, the tensile resistance of concrete in the tension zone is conservatively ignored.

$$h_n = \frac{0.85 A_c f_{cd}}{2 \times 0.85 f_{cd} b_c + 4 t_{sz} (2 f_{sd} - 0.85 f_{cd}) + 2 t_w (2 f_{ed} - 0.85 f_{cd})}$$
$$= \frac{0.85 \times 144400 \times 33.3}{2 \times 0.85 \times 33.3 \times 400 + 4 \times 2 \times (2 \times 400 - 0.85 \times 33.3) + 2 \times 12.8 \times (2 \times 355 - 0.85 \times 33.3)}$$
$$= 88.7 \text{ mm}$$

$h_n = 88.7 \text{ mm} < h/2 - t_f = 266.7/2 - 20.5 = 112.85 \text{ mm}$, thus, the neutral axial lies in the web of the encased section.

The plastic modulus of the equivalent tube of reinforcements, encased section, and concrete in the height of $2h_n$, bending about centreline of the cross-section are calculated as:

$$W_{sy,n} = 2 t_{sz} h_n^2 = 2 \times 2 \times 88.7^2 \times 10^{-3} = 31.5 \text{ cm}^3$$

$$W_{ey,n} = t_w h_n^2 = 12.8 \times 88.7^2 \times 10^{-3} = 100.7 \text{ cm}^3$$

$$W_{cy,n} = (b_c - 2t_{sz} - t_w) h_n^2 = (400 - 2 \times 2 - 12.8) \times 88.7^2 \times 10^{-3} = 3015 \text{ cm}^3$$

Taking moment about the centreline of the cross-section, the plastic bending resistance is determined from:

$$M_{pl,Rd} = (W_{sy} - W_{sy,n}) f_{sd} + (W_{ey} - W_{ey,n}) f_{ed} + 0.5 (W_{cy} - W_{cy,n}) f_{cd}$$
$$= \left[(338 - 31.5) \times 400 + (1469 - 100.7) \times 355 + 0.5 \times (14192 - 3015) \times 33.3\right] \times 10^{-3}$$
$$= 767 \text{ kN} \cdot \text{m}$$

It should be noted that the plastic bending resistance can be calculated by taking moment about either line on the cross-section parallel to the y-y axis, as long as the aforementioned plastic modulus are determined according to the referred line.

3) Point C ($M_{pl,Rd}$, $N_{pm,Rd}$):

The cross-section is under partial compression but axial force is resultant from the compressive stresses. The axial force is equal to the compression capacities of concrete in the compression zone and steel sections within the height of $2h_n$. It is mentioned above that the compression capacity of steel sections within the height of $2h_n$ is equal to the compression capacity of concrete in the compression zone and out of the height of $2h_n$. Thus, the axial force is actually the full cross-sectional compression capacity of concrete and determined from:

$$N_{pm,Rd} = 0.85 A_c f_{cd} = 0.85 \times 144400 \times 33.3 \times 10^{-3} = 4091\,\text{kN}$$

4) Point D ($M_{max,Rd}$, $N_{pm,Rd}/2$):

The maximum plastic moment resistance $M_{max,Rd}$ is calculated when the h_n is equal to 0.

$$\begin{aligned} M_{\max,Rd} &= W_{sy} f_{sd} + W_{ey} f_{ed} + 0.5 \times 0.85 W_{cy} f_{cd} \\ &= [338 \times 435 + 1469 \times 355 + 0.5 \times 0.85 \times 14192 \times 33.3] \times 10^{-3} \\ &= 870\text{kN} \cdot \text{m} \end{aligned}$$

- Steel contribution ratio

$$\begin{aligned} \delta &= (A_e f_{ed}) / N_{pl,Rd} \\ &= (13500 \times 355) \times 10^{-3} / 9733 = 0.492 < 0.9 \end{aligned}$$

- Check for resistance of column in axial compression

$$\frac{N_{Ed}}{\chi N_{pl,Rd}} = \frac{6000}{0.766 \times 9806} = 0.805 < 1.0$$

Thus, the buckling resistance under axial compression is adequate!

- Check for resistance of column in combined compression and uniaxial bending

For the determination of the internal forces considering second-order effect, the design value of effective flexural stiffness should be calculated as following with long-term effect included.

$$(EI)_{eff,II} = K_0(E_s I_{sy} + E_e I_{ey} + K_{e,II} E_{cm} I_{cy})$$

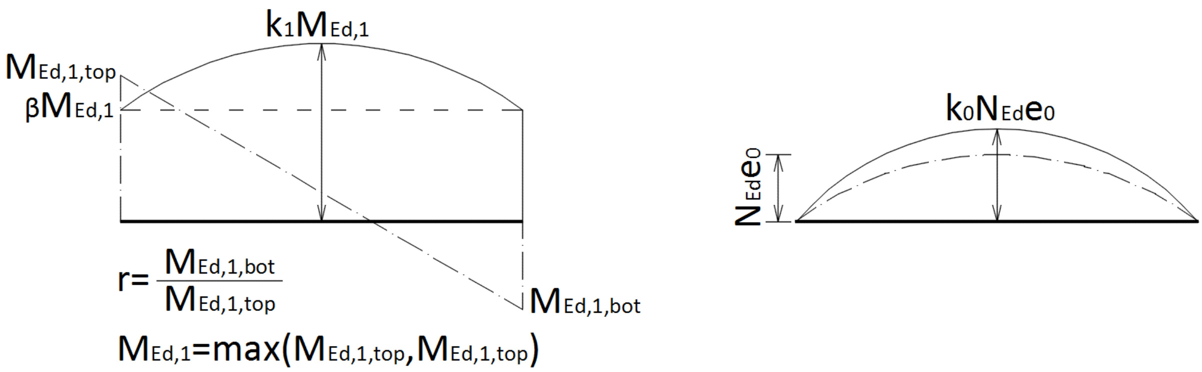
$$= 0.9 \times [210 \times (5115 + 17343) + 0.5 \times 20.7 \times 190876] \times 10^4$$

$$= 6.02 \times 10^{10}\ \text{kN} \cdot \text{mm}^2$$

Thus, the critical normal force about y-y axis with effective length taken as the system length of column is determined from:

$$N_{cr,eff} = \frac{\pi^2 (EI)_{eff,II}}{L^2} = \frac{\pi^2 \times 6.02 \times 10^{10}}{4000^2} = 37154\ \text{kN}$$

The second-order effect should be considered for both moments from first-order analysis and moment from imperfection as following:



(a) Moment from first-order analysis (b) Moment from imperfection

According to buckling curve "b", the initial imperfection about y-y axial is:

$$e_{0,y} = L/200 = 4000/200 = 20\ \text{mm}$$

Accordingly, the bending moment by the initial imperfection is determined as:

$$M_0 = N_{Ed} e_{0,y} = 6000 \times 20 / 1000 = 120\ \text{kN} \cdot \text{m}$$

According to the moment diagram by the initial imperfection, the factor β_0 for determination of moment to second-order effect is equal to 1.0. Thus, the amplification factor for the moment by the imperfection is calculated from:

$$k_0 = \frac{\beta_0}{1 - N_{Ed}/N_{cr,eff}} = \frac{1.0}{1 - 6000/37154} = 1.193$$

According to the first-order design moment diagram, the ratio of end moments is calculated as:

$$r = M_{t,y} / M_{b,y} = -200/400 = -0.5$$

Thus, the factor β1 for determination of moment to second-order effect is determined:

$$\beta_1 = \max(0.66 + 0.44r, 0.44) = \max(0.66 + 0.44 \times (-0.5), 0.44) = 0.44$$

Thus, the amplification factor for the moment by the imperfection is calculated from:

$$k_1 = \frac{\beta_1}{1 - N_{Ed} / N_{cr,eff}} = \frac{0.44}{1 - 6000/37154} = 0.525$$

Thus, the design moment, considering second-order effect, is calculated as:

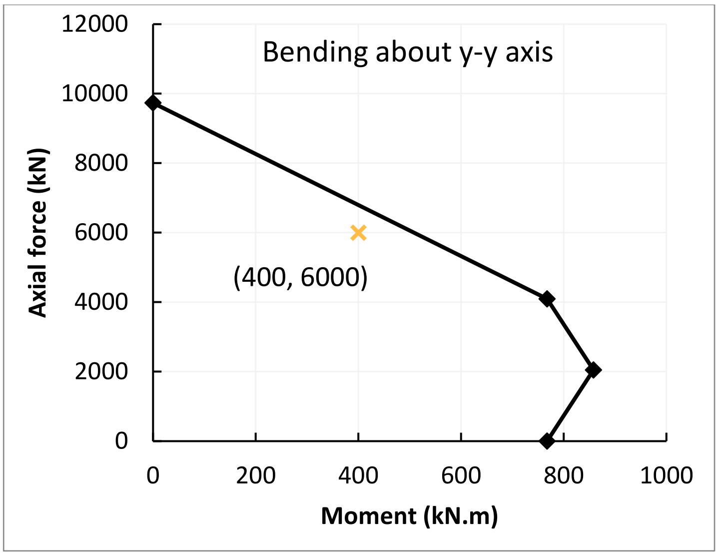
$$\begin{aligned} M_{Ed} &= \max\left[k_0 M_0 + k_1 \max(|M_{t,y}|, |M_{b,y}|), \max(|M_{t,y}|, |M_{b,y}|)\right] \\ &= \max\left[1.193 \times 120 + 0.525 \times \max(|-200|, |400|), \max(|-200|, |400|)\right] = 400\ \mathrm{kN \cdot m} \end{aligned}$$

In this case, the second-order effect is not significant and the maximum end moment is taken as the design moment. For $N_{Ed} > N_{pm,Rd} = 4091\ kN$, the value for determining the plastic bending resistance $M_{pl,N,Rd}$ taking into account the normal force N_{Ed} is calculated from:

$$\mu_d = \frac{N_{pl,Rd} - N_{Ed}}{N_{pl,Rd} - N_{pm,Rd}} = \frac{9806 - 6000}{9806 - 4091} = 0.662$$

$$\frac{M_{Ed}}{M_{pl,N,Rd}} = \frac{M_{Ed}}{\mu_d M_{pl,Rd}} = \frac{400}{0.662 \times 767} = 0.788 < \alpha_M = 0.9$$

Thus, the resistance for combined axial compression and uniaxial bending is adequate. The external design force and bending moment, and M-N interaction curve are plotted:



4.3 UC 254x254x107 S355 sections with C90/105 concrete and G500 reinforcements

Effective compressive strength and modulus of elasticity are taken from Table 2.9 and Table 2.10

$f_{ck} = 72 \ \text{N/mm}^2; \ E_{cm} = 41.1 \ \text{GPa}$

$f_{cd} = f_{ck}/\gamma_c = 72/1.5 = 48 \ \text{N/mm}^2$

$f_{cm} = f_{ck} + 8 = 72 + 8 = 80 \ \text{N/mm}^2$

Creep coefficient could be similarly determined: $\varphi_t = 1.20$

$$E_{c,eff} = E_{cm}\frac{1}{1 + (N_{G,Ed}/N_{Ed})\phi_t} = \frac{41.1}{1 + (4000/6000) \times 1.20} = 22.83\text{GPa}$$

$$(EI)_{eff,y} = E_s I_{sy} + E_e I_{ey} + 0.6 E_{c,eff} I_{cy}$$
$$= [210 \times (5115 + 17343) + 0.6 \times 22.83 \times 190876] \times 10^4$$
$$= 7.33 \times 10^{10} kN.mm^2$$

$$(EI)_{eff,z} = E_s I_{sz} + E_e I_{ez} + 0.6 E_{c,eff} I_{cz}$$
$$= [210 \times (5115 + 5926) + 0.6 \times 22.83 \times 202292] \times 10^4$$
$$= 5.09 \times 10^{10} kN.mm^2$$

$$N_{cr,y} = \frac{\pi^2 (EI)_{eff,y}}{L_{eff}^2} = \frac{\pi^2 \times 7.33 \times 10^{10}}{4000^2} = 45215\text{kN}$$

$$N_{cr,z} = \frac{\pi^2 (EI)_{eff,z}}{L_{eff}^2} = \frac{\pi^2 \times 5.09 \times 10^{10}}{4000^2} = 31398\text{kN}$$

$$N_{pl,Rk} = A_s f_{sk} + A_e f_{ek} + 0.85 A_c f_{ck}$$

$$= 2120 \times 500 + 13500 \times 355 + 0.85 \times 144400 \times 72 = 14690 \text{ kN}$$

$$\overline{\lambda_y} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{14690}{45215}} = 0.570$$

$$\overline{\lambda_z} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{14690}{31398}} = 0.684$$

For a fully encased steel column, the buckling curve about major axis is "b", and about minor axis is "c". Thus, the imperfection factor is $\alpha = 0.34$ and $\alpha = 0.49$, respectively.

$$\Phi_y = 0.5\left[1 + \alpha_y(\overline{\lambda_y} - 0.2) + \overline{\lambda_y}^2\right] = 0.5[1 + 0.34 \times (0.570 - 0.2) + 0.570^2] = 0.725$$

$$\chi_y = min\left(\frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \overline{\lambda}_y^2}}, 1.0\right) = min\left(\frac{1}{0.725 + \sqrt{0.725^2 - 0.570^2}}, 1.0\right) = 0.852$$

$$\Phi_z = 0.5\left[1 + \alpha_z(\overline{\lambda_z} - 0.2) + \overline{\lambda_z}^2\right] = 0.5[1 + 0.49 \times (0.684 - 0.2) + 0.684^2] = 0.853$$

$$\chi_z = min\left(\frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \overline{\lambda}_z^2}}, 1.0\right) = min\left(\frac{1}{0.853 + \sqrt{0.853^2 - 0.684^2}}, 1.0\right) = 0.734$$

$$\chi = min(\chi_y, \chi_z) = min(0.852, 0.734) = 0.734$$

Point A (0, $N_{pl,Rd}$):

$$N_{pl,Rd} = A_s f_{sd} + A_e f_{ed} + 0.85 A_c f_{cd}$$

$$= 2120 \times 435 + 13500 \times 355 + 0.85 \times 144400 \times 48 = 11606 \text{ kN}$$

Buckling resistance: $\chi N_{pl,Rd} = 0.734 \times 11606 = 8519$ kN

Point B ($M_{pl,Rd}$, 0):

$$h_n = \frac{0.85A_c f_{cd}}{2\times 0.85 f_{cd} b_c + 4t_{sz}(2f_{sd} - 0.85f_{cd}) + 2t_w(2f_{ed} - 0.85f_{cd})}$$

$$= \frac{0.85\times 144400\times 48}{2\times 0.85\times 48\times 400 + 4\times 2\times (2\times 400 - 0.85\times 48) + 2\times 12.8\times (2\times 355 - 0.85\times 48)}$$

$$= 105.8\ \text{mm}$$

$h_n = 105.8\ \text{mm} < h/2 - t_f = 266.7/2 - 20.5 = 112.85\ \text{mm}$, thus, the neutral axial also lies in the web of the encased section.

$$W_{sy,n} = 2t_{sz}h_n^2 = 2\times 2\times 105.8^2\times 10^{-3} = 44.8\ \text{cm}^3$$

$$W_{ey,n} = t_w h_n^2 = 12.8\times 105.8^2\times 10^{-3} = 143.3\ \text{cm}^3$$

$$W_{cy,n} = (b_c - 2t_{sz} - t_w)h_n^2 = (400 - 2\times 2 - 12.8)\times 105.8^2\times 10^{-3} = 4289\ \text{cm}^3$$

$$M_{pl,Rd} = (W_{sy} - W_{sy,n})f_{sd} + (W_{ey} - W_{ey,n})f_{ed} + 0.5(W_{cy} - W_{cy,n})f_{cd}$$

$$= [(338 - 44.8)\times 400 + (1469 - 143.3)\times 355 + 0.5\times (14192 - 4289)\times 48]\times 10^{-3}$$

$$= 791\ \text{kN}\cdot\text{m}$$

Point C ($M_{pl,Rd}$, $N_{pm,Rd}$):

$$N_{pm,Rd} = 0.85A_c f_{cd} = 0.85\times 144400\times 48\times 10^{-3} = 5891\ \text{kN}$$

Point D ($M_{max,Rd}$, $N_{pm,Rd}/2$):

$$M_{\max,Rd} = W_{sy}f_{sd} + W_{ey}f_{ed} + 0.5\times 0.85W_{cy}f_{cd}$$

$$= [338\times 435 + 1469\times 355 + 0.5\times 0.85\times 14192\times 48]\times 10^{-3}$$

$$= 958\ \text{kN}\cdot\text{m}$$

Steel contribution ratio:

$$\delta = (A_e f_{ed})/N_{pl,Rd}$$

$$= (13500\times 355)\times 10^{-3}/11606 = 0.416 < 0.9$$

4.4 UC 254x254x107 S500 sections with C50/60 concrete and G500 reinforcements

$$N_{pl,Rk} = A_s f_{sk} + A_e f_{ek} + 0.85 A_c f_{ck}$$

$$= 2120 \times 500 + 13500 \times 500 + 0.85 \times 144400 \times 33.3 = 13948 \text{ kN}$$

$$\overline{\lambda_y} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,y}}} = \sqrt{\frac{13948}{45215}} = 0.555$$

$$\overline{\lambda_z} = \sqrt{\frac{N_{pl,Rk}}{N_{cr,z}}} = \sqrt{\frac{13948}{31398}} = 0.666$$

For a fully encased steel column, the buckling curve about major axis is "b", and about minor axis is "c". Thus, the imperfection factor is $\alpha = 0.34$ and $\alpha = 0.49$, respectively.

$$\Phi_y = 0.5\left[1 + \alpha_y(\overline{\lambda_y} - 0.2) + \overline{\lambda_y}^2\right] = 0.5[1 + 0.34 \times (0.555 - 0.2) + 0.555^2] = 0.714$$

$$\chi_y = min\left(\frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \overline{\lambda_y^2}}}, 1.0\right) = min\left(\frac{1}{0.714 + \sqrt{0.714^2 - 0.555^2}}, 1.0\right) = 0.860$$

$$\Phi_z = 0.5\left[1 + \alpha_z(\overline{\lambda_z} - 0.2) + \overline{\lambda_z}^2\right] = 0.5[1 + 0.49 \times (0.666 - 0.2) + 0.666^2] = 0.836$$

$$\chi_z = min\left(\frac{1}{\Phi_z + \sqrt{\Phi_z^2 - \overline{\lambda_z^2}}}, 1.0\right) = min\left(\frac{1}{0.836 + \sqrt{0.836^2 - 0.666^2}}, 1.0\right) = 0.746$$

$$\chi = min(\chi_y, \chi_z) = min(0.860, 0.746) = 0.746$$

Point A (0, $N_{pl,Rd}$):

$$N_{pl,Rd} = A_s f_{sd} + A_e f_{ed} + 0.85 A_c f_{cd}$$

$$= 2120 \times 435 + 13500 \times 500 + 0.85 \times 144400 \times 33.3 = 11764 \text{ kN}$$

Buckling resistance: $\chi N_{pl,Rd} = 0.746 \times 11764 = 8776\text{kN}$

Point B ($M_{pl,Rd}$, 0):

$$h_n = \frac{0.85 A_c f_{cd}}{2\times 0.85 f_{cd} b_c + 4t_{sz}(2f_{sd} - 0.85 f_{cd}) + 2t_w(2f_{ed} - 0.85 f_{cd})}$$

$$= \frac{0.85\times 144400\times 33.3}{2\times 0.85\times 33.3\times 400 + 4\times 2\times(2\times 435 - 0.85\times 33.3) + 2\times 12.8\times(2\times 500 - 0.85\times 33.3)}$$

$$= 76.4\ \text{mm}$$

The neutral axial also lies in the web of the encased section.

$$W_{sy,n} = 2t_{sz}h_n^2 = 2\times 2\times 76.4^2\times 10^{-3} = 23.3\ \text{cm}^3$$

$$W_{ey,n} = t_w h_n^2 = 12.8\times 76.4^2\times 10^{-3} = 74.7\ \text{cm}^3$$

$$W_{cy,n} = (b_c - 2t_{sz} - t_w)h_n^2 = (400 - 2\times 2 - 12.8)\times 76.4^2\times 10^{-3} = 2237\ \text{cm}^3$$

$$M_{pl,Rd} = (W_{sy} - W_{sy,n})f_{sd} + (W_{ey} - W_{ey,n})f_{ed} + 0.5(W_{cy} - W_{cy,n})f_{cd}$$

$$= \left[(338 - 23.3)\times 435 + (1469 - 74.7)\times 500 + 0.5\times(14192 - 2237)\times 33.3\right]\times 10^{-3}$$

$$= 1006\ \text{kN}\cdot\text{m}$$

Point C ($M_{pl,Rd}$, $N_{pm,Rd}$):

$$N_{pm,Rd} = 0.85 A_c f_{cd} = 0.85\times 144400\times 33.3\times 10^{-3} = 4091\ \text{kN}$$

Point D ($M_{max,Rd}$, $N_{pm,Rd}/2$):

$$M_{\max,Rd} = W_{sy} f_{sd} + W_{ey} f_{ed} + 0.5\times 0.85 W_{cy} f_{cd}$$

$$= \left[338\times 435 + 1469\times 500 + 0.5\times 0.85\times 14192\times 33.3\right]\times 10^{-3}$$

$$= 1083\ \text{kN}\cdot\text{m}$$

Steel contribution ratio:

$$\delta = (A_e f_{ed})/N_{pl,Rd}$$

$$= (13500\times 500)\times 10^{-3}/11764 = 0.577 < 0.9$$

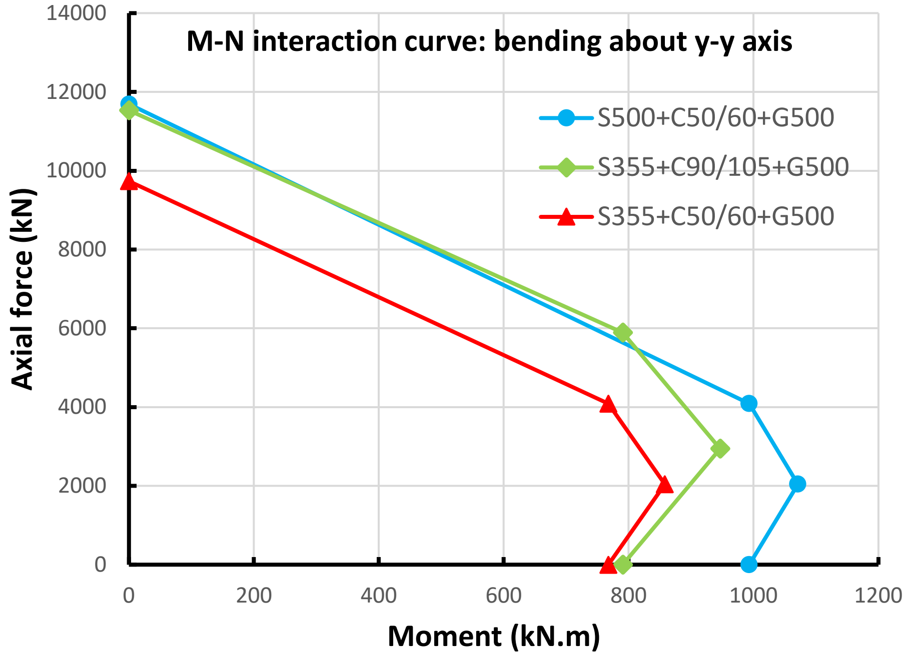
4.5 Comparison and summary

The design resistances are compared for the aforementioned three composite sections. The composite section with encased steel section of S355, concrete of C50/60, and reinforcing steel of G500 is referred to for comparison.

By use of high strength concrete C90/105 replacing normal strength concrete C50/60, the axial buckling resistance ($\chi N_{pl,Rd}$) of the CES column is improved by 15.7%, and the increase of moment resistances ($M_{pl,Rd}$ and $M_{max,Rd}$) are 3.1% and 10.3%, respectively.

By use of steel S500 replacing S355, the axial buckling resistance is improved by 15.4%, and the increase of moment resistance is as high as 29.5% and 24.8%.

Material grades (Steel+ Concrete+ Rebars)	**Steel contribution ratios**	**Design resistances**			
		$\chi N_{pl,Rd}$	$N_{pm,Rd}$	$M_{pl,Rd}$	$M_{max,Rd}$
S355+C50/60+G500	0.492	0	0	0	0
S355+C90/105+G500	0.416	15.7%	44.0%	3.1%	10.3%
S500+C50/60+G500	0.577	15.4%	0	29.5%	24.8%



Through this study, it could be concluded that for CES columns with small eccentricities (large axial load and small bending moments), it is beneficial to use high strength concrete; whereas for CES columns with smaller axial load and higher bending moments, the use of high strength steel is beneficial.

5 Example 5

5.1 General

In Example 5, the buckling resistance of a Pre-fabricated Steel Reinforced Concrete (PSRC) column subject to axial compression and biaxial bending is determined. This example illustrates a manual calculation approach. A more detailed analysis can also be performed using structural analysis software if needed.

The PSRC column consists of battened truss built with equal angle sections of encased in concrete. The dimensions of the PSRC column are shown in Figure 16. The dimensions of the built-up steel member of the PSRC column are show in Figure 17.

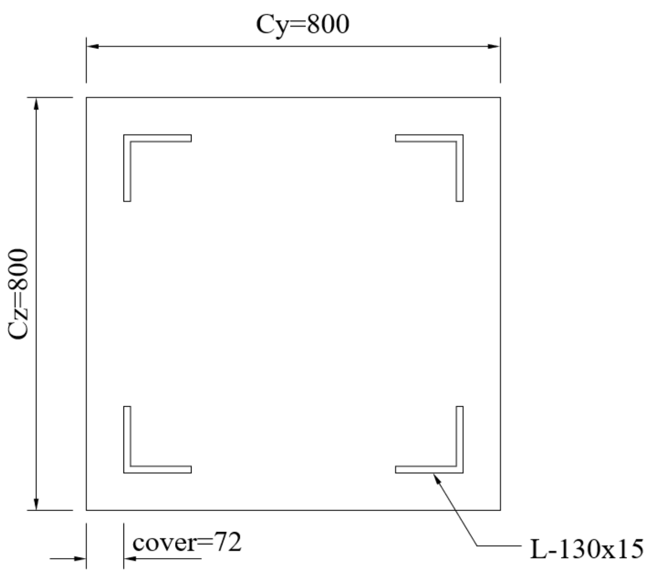


Figure 16 Cross-sectional dimensions of PSRC column in Example 5

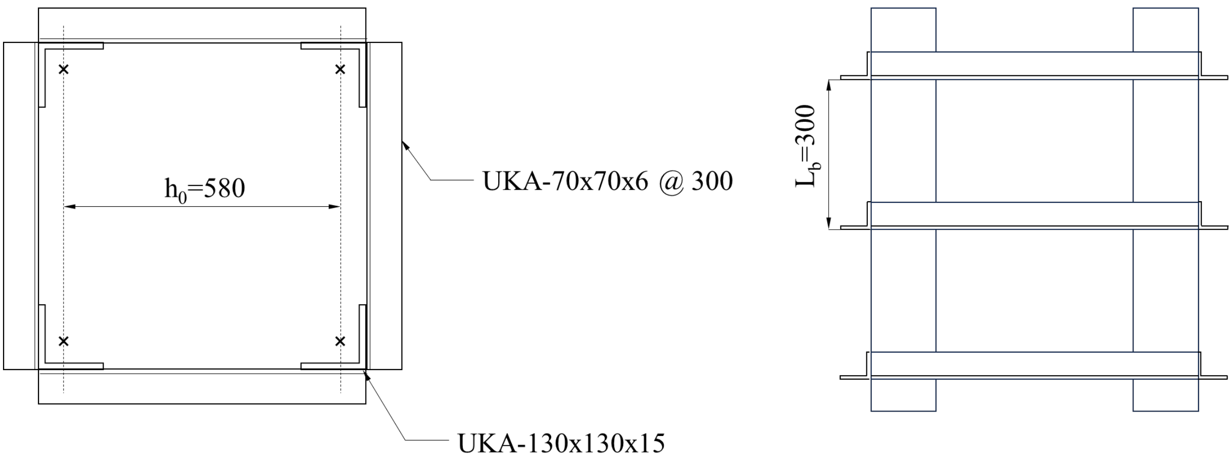


Figure 17 Dimensions of steel built-up member of PSRC column in Example 5

To evaluate the resistance, the following steel and concrete are used:

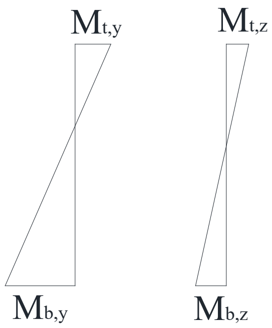
a) 4 x UKA 130x130x15 – S355 steel sections with C50/60 concrete under service loads

b) 4 x UKA 130x130x15 – S355 steel sections with UKA 70x70x6 S275 steel sections – 300mm c/c spg under construction loads

5.2 4 x UKA 130x130x15 – S355 steel sections with C50/60 concrete under service loads

- Design Parameters:

Concrete	C50/60, f_{ck}=50 N/mm^2
Embedded steel angle section	Grade S355, f_{ek}=355 N/mm^2
Column system length	L=8000 mm
Effective length	L_{eff} =8000 mm
Total design axial load	N_{Ed}=6000 kN
Design axial load that is permanent	$N_{G,Ed}$=4000 kN



Design moment at bottom around y-y axis	$M_{b,y}$=900 kN.m
Design moment at top around y-y axis	$M_{t,y}$= - 300 kN.m
Design moment at bottom around z-z axis	$M_{b,y}$= 700 kN.m
Design moment at top around z-z axis	$M_{t,y}$= - 400 kN.m
Design shear force along y-y axis	V_y= 100 kN
Design shear force along z-z axis	V_z= 100 kN

- Material

Concrete	C50/60, f_{ck}=50 N/mm^2
Angle sections	Grade S355, f_y=355 N/mm^2

- Design strength and modulus of material

$f_{yd} = f_y/\gamma_a = 355/1.0 = 355\ \text{N/mm}^2$

$f_{cd} = f_{ck}/\gamma_c = 50/1.5 = 33.3\ \text{N/mm}^2$

$f_{cm} = f_{ck} + 8 = 58\ \text{N/mm}^2$

$E_s = 210\ \text{GPa}$

$E_{cm} = 22(f_{cm}/10)^{0.3} = 22(58/10)^{0.3} = 37.3\ \text{GPa}$

- Cross sectional areas

$A = C_yC_z = 800 \times 800 = 640000\ \text{mm}^2$

$A_s = 3675 \times 4 = 14700\ \text{mm}^2$

$A_c = A - A_s = 640000 - 14700 = 625300\ \text{mm}^2$

- Second moment of areas

$I_s = (5680000 + 3675 \times 290.4^2) \times 4/10^4 = 126240\ \text{cm}^4$

$I_c = I - I_s = 3413333 - 126240 = 3287093\ \text{cm}^4$

- Plastic modulus

$W_y = C_yC_z^2/4 = 800^3/4 = 128000\ \text{cm}^3$

$W_{sy} = 2(15 \times 328^2 - 15 \times 198^2 + 115 \times 328^2 - 115 \times 313^2) = 4263\ \text{cm}^3$

$W_{cy} = W_y - W_{sy} = 128000 - 4263 = 123737\ \text{cm}^3$

- Concrete cover check

According to EN-1994-1-1:2004 cl. 6.7.5.1(2), the concrete cover should be not less than 400mm, nor less than one-sixth of the breadth of the angle.

$\text{cover} = 72\text{mm} > \max(40, 130/6) = 40\ \text{mm}$

- Long-term effect

  Age of concrete at loading in day: $t_0 = 28$

  Age of concrete at the moment considered in days: $t = \infty$

  Relative humidity of ambient environment: $RH = 50\%$

  Perimeter of concrete section: $u = 2C_y + 2C_z = 3200\ \text{mm}$

  Notional size of concrete section: $h_0 = 2A_c/u = 2 \times 625300/3200 = 391\ \text{mm}$

  Coefficient: $\alpha_1 = (35/f_{cm})^{0.7} = 0.70$

  Coefficient: $\alpha_2 = (35/f_{cm})^{0.2} = 0.90$

  Coefficient: $\alpha_3 = (35/f_{cm})^{0.5} = 0.78$

  Temperature adjusted age of concrete: $t_{0,T} = \exp(-(4000/[273 + T]) - 13.65) \times t_0 = 27.9$

  Modified age of loading: $t_0 = t_{0,T} \times \left(\frac{9}{2+t_{0,T}^{1.2}} + 1\right)^{\alpha} = 24.1 \geq 0.5.$

  Factor: $\varphi_{RH} = \left(1 + \frac{1-RH/100}{0.1\sqrt[3]{h_0}}\alpha_1\right)\alpha_2 = \left(1 + \frac{1-50/100}{0.1\sqrt[3]{391}}0.70\right) \times 0.90 = 1.33$

  Factor: $\beta(f_{cm}) = 16.8/\sqrt{f_{cm}} = 16.8/\sqrt{58} = 2.21$

  Factor: $\beta(t_0) = 1/(0.1 + t_0^{0.2}) = 1/(0.1 + 24.1^{0.2}) = 0.50$

  Factor: $\varphi_0 = \varphi_{RH}\beta(f_{cm})\beta(t_0) = 1.33 \times 2.21 \times 0.50 = 1.47$

  Factor: $\beta_H = 1.5[1 + (0.012RH)^{18}]h_0 + 250\alpha_3 = 1.5[1 + (0.012 \times 50)^{18}] \times 391 + 250 \times 0.78 = 782$

  Factor: $\beta_c(t, t_0) = \left(\frac{t-t_0}{\beta_H+t-t_0}\right)^{0.3} = 1$

  Creep coefficient: $\varphi_t = \varphi_0\beta_c(t, t_0) = 1.47 \times 1 = 1.47$

- Elastic modulus of concrete considering long-term effect

  $$E_{c,eff} = E_{cm}\frac{1}{1+(N_{G,Ed}/N_{Ed})\varphi_t} = \frac{37.3}{1+(4000/6000)\times 1.47} = 18.84\ \text{GPa}$$

- Effective flexural stiffness of cross-section

$$(EI)_{eff} = E_s I_s + 0.6E_{c,eff} I_c = [210 \times 126240 + 0.6 \times 18.75 \times 3287093] \times 10^4 = 63.48 \times 10^{10}\ \mathrm{kN \cdot mm^2}$$

- Steel contribution ratio

$$\delta = (A_s f_{yd})/N_{pl,Rd} = (14700 \times 355) \times 10^{-3}/22935 = 0.228 < 0.9$$

- Elastic critical Euler buckling resistance

$$N_{cr} = \frac{\pi^2 (EI)_{eff}}{L_{eff}^2} = \frac{\pi^2 \times 63.48 \times 10^{10}}{8000^2} = 97889\ \mathrm{kN}$$

- Characteristic plastic resistance of cross-section

$$N_{pl,Rk} = A_s f_y + 0.85 A_c f_{ck} = 14700 \times 355 + 0.85 \times 625300 \times 50 = 31794\ \mathrm{kN}$$

- Relative slenderness ratio

$$\overline{\lambda_y} = \sqrt{\frac{N_{pl,Rk}}{N_{cr}}} = \sqrt{\frac{31794}{97889}} = 0.570$$

- Buckling curves and buckling reduction factors

For a fully encased steel column, the buckling curve about major axis is "b". The imperfection factor is $\alpha_y = 0.34$.

$$\Phi_y = 0.5\left[1 + \alpha_y\left(\overline{\lambda_y} - 0.2\right) + \overline{\lambda_y}^2\right]$$

$$= 0.5[1 + 0.34 \times (0.570 - 0.2) + 0.570^2]$$

$$= 0.725$$

$$\chi_y = \min\left(\frac{1}{\Phi_y + \sqrt{\Phi_y^2 - \overline{\lambda_y}^2}}, 1.0\right) = \min\left(\frac{1}{0.725 + \sqrt{0.725^2 - 0.570^2}}, 1.0\right) = 0.852$$

$$\chi = \chi_y = 0.852$$

- Simplified Interaction Curve

1) Point A (0, $N_{pl,Rd}$):

Full cross-section is under uniform compression. No bending moment is resultant from the compressive stresses on the cross-section.

$$N_{pl,Rd} = A_s f_{yd} + 0.85 A_c f_{cd}$$

$$= 14700 \times 355 + 0.85 \times 625300 \times 33.3$$

$$= 22935 \text{ kN}$$

2) Point B ($M_{pl,Rd}$, 0):

The cross-section is under partial compression and no axial force in formed. Assuming the neutral axis lies in the web of angle ($198 \text{ mm} \leq h_n \leq 313 \text{ mm}$), the height of neutral axis is calculated based on the force equilibrium between the tensile capacity of steel sections within the height of $2h_n$ is equal to the compression resistance of concrete in the compression zone. Unless other stated, the tensile resistance of concrete in the tension zone is conservatively ignored.

$$h_n = \frac{0.85 f_{cd} A_c - 0.85 f_{cd} \times 11880 + 2 f_{yd} \times 11880}{0.85 f_{cd} \times 1540 + 2 f_{yd} \times 60}$$

$$= \frac{0.85 \times 33.3 \times 625300 - 0.85 \times 33.3 \times 11880 + 2 \times 355 \times 11880}{0.85 \times 33.3 \times 1540 + 2 \times 355 \times 60}$$

$$= 299.3 \text{ mm}$$

$h_n = 299.3 \text{ mm}$ is larger than 198 mm and smaller than 313 mm, thus, the neutral axial lies in the web of the steel angle.

The plastic modulus of steel angles and concrete in the height of $2h_n$, bending about centreline of the cross-section are calculated as:

$$W_{sy,n} = 2(15h_n^2 - 15 \times 198^2) \times 10^{-3} = 1511 \text{ cm}^3$$

$$W_{cy,n} = \left[(C_y - 30)h_n^2 + 2 \times 15 \times 198^2\right] \times 10^{-3} = 70153 \text{ cm}^3$$

Taking moment about the centreline of the cross-section, the plastic bending resistance is determined from:

$$M_{pl,Rd} = (W_{sy} - W_{sy,n}) f_{yd} + 0.5(W_{cy} - W_{cy,n}) f_{cd}$$

$= [(4260 - 1511) \times 355 + 0.5 \times (123737 - 70153) \times 33.3] \times 10^{-3}$

$= 1868 \text{ kN} \cdot \text{m}$

It should be noted that the plastic bending resistance can be calculated by taking moment about either line on cross-section parallel to the y-y axis, as long as the aforementioned plastic modulus are determined according to the referred line.

3) Point C ($M_{pl,Rd}$, $N_{pm,Rd}$):

The cross-section is under partial compression but axial force is resultant from the compressive stresses. The axial force is equal to the compression capacities of concrete in the compression zone and steel sections within the height of $2h_n$. It is mentioned about that the compression capacity of steel section within the height of $2h_n$ is equal to the compression capacity of concrete in the compression zone and out of the height of $2h_n$. Thus, the axial force is actually the full cross-sectional compression capacity of concrete and determine from:

$N_{pm,Rd} = 0.85A_c f_{cd} = 0.85 \times 625300 \times 33.3 \times 10^{-3} = 17699 \text{ kN}$

4) Point D ($M_{max,Rd}$, $N_{pm,Rd}/2$):

The maximum plastic moment resistance $M_{max,Rd}$ is calculated when the h_n is equal to 0.

$M_{max,Rd} = W_{sy} f_{yd} + 0.5 \times 0.85 W_{cy} f_{cd}$

$= [4260 \times 355 + 0.5 \times 0.85 \times 123737 \times 33.3] \times 10^{-3}$

$= 3263 \text{ kN} \cdot \text{m}$

- Check for resistance of column in axial compression

  $N_{Ed}/\chi N_{pl,Rd} = 6000/(0.852 \times 22935) = 0.307 < 1.0$

  Thus, the buckling resistance under axial compression is adequate!

- Check for resistance of column in combined compression and biaxial bending

  For determination of the internal forces considering second-order effect, the design value of effective flexural stiffness should be calculated with long-term effect included.

$$(EI)_{eff,II} = K_0\left(E_s I_s + K_{e,II} E_{c,eff} I_c\right)$$

$$= 0.9 \times [210 \times 126240 + 0.5 \times 18.84 \times 3287093] \times 10^4$$

$$= 63.48 \times 10^{10}\ \mathrm{kN \cdot mm^2}$$

Thus, the critical normal force about y-y axis with effective length taken as the system length of column is determined from:

$$N_{cr,eff} = \frac{\pi^2 (EI)_{eff,II}}{L^2} = \frac{\pi^2 \times 63.48 \times 10^{10}}{8000} = 97889\ \mathrm{kN}$$

The second-order effect should be considered for both moments from first-order analysis and moment from imperfection as following:

According to buckling curve “b”, the initial imperfection about y-y axis and z-z axis are:

$$e_0 = L/200 = 8000/200 = 40\ \mathrm{mm}$$

Accordingly, the bending moment by the initial imperfection is determined as:

$$M_0 = N_{Ed} e_0 = 6000 \times 40/1000 = 240\ \mathrm{kN \cdot m}$$

According to the moment diagram by the initial imperfection, the factor β_0 for determination of moment to second-order effect is equal to 1.0. Thus, the amplification factor for the moment by the imperfection is calculated from:

$$k_0 = \frac{\beta_0}{1 - N_{Ed}/N_{cr,eff}} = \frac{1.0}{1 - 6000/97889} = 1.065$$

According to the first-order design moment diagram, the ratio of end moment is calculated as:

$$r_y = M_{t,y}/M_{b,y} = -300/900 = -0.33$$

$$r_z = M_{t,z}/M_{b,z} = -400/700 = -0.57$$

Thus, the factor β_1 for determination of moment to second-order effect is determined:

$$\beta_{1,y} = \max\left(0.66 + 0.44 r_y, 0.44\right) = \max(0.66 + 0.44 \times (-0.33), 0.44) = 0.51$$

$$\beta_{1,z} = \max(0.66 + 0.44 r_z, 0.44) = \max(0.66 + 0.44 \times (-0.57), 0.44) = 0.44$$

Thus, the amplification factor for the moment by the imperfection is calculated from:

$$k_{1,y} = \frac{\beta_{1,y}}{1 - N_{Ed}/N_{cr,eff}} = \frac{0.51}{1 - 6000/97889} = 0.543$$

$$k_{1,z} = \frac{\beta_{1,z}}{1 - N_{Ed}/N_{cr,eff}} = \frac{0.44}{1 - 6000/97889} = 0.469$$

Thus, the design moment, considering second-order effect, is calculated as:

$$M_{y,Ed} = \max\left[k_0 M_0 + k_{1,y} \max\left(\left|M_{t,y}\right|, \left|M_{b,y}\right|\right), \max\left(\left|M_{t,y}\right|, \left|M_{b,y}\right|\right)\right]$$

$$= \max[1.065 \times 240 + 0.543 \times max(|-300|, |900|), max(|-300|, |900|)]$$

$$= 900 \text{ kN} \cdot \text{m}$$

$$M_{z,Ed} = \max\left[k_0 M_0 + k_{1,z} \max\left(\left|M_{t,z}\right|, \left|M_{b,z}\right|\right), \max\left(\left|M_{t,z}\right|, \left|M_{b,z}\right|\right)\right]$$

$$= \max[1.065 \times 240 + 0.469 \times max(|-400|, |700|), max(|-400|, |700|)]$$

$$= 700 \text{ kN} \cdot \text{m}$$

In this case, the second-order effect is not significant and the maximum end moment is taken as the design moment. For $N_{Ed} < N_{pm,Rd}/2 = 8849.5 \text{ kN}$, the value for determining the plastic bending resistance $M_{pl,N,y,Rd}$ and $M_{pl,N,z,Rd}$ taking into account the normal force N_{Ed} is calculated from:

$$M_{pl,N,y,Rd} = M_{pl,N,z,Rd} = M_{pl,Rd} + \left(M_{max,Rd} - M_{pl,Rd}\right)\frac{N_{Ed}}{N_{pm,Rd}/2}$$

$$= 1868 + (3263 - 1868) \times \frac{6000}{8849.5}$$

$$= 2814$$

$$\frac{M_{y,Ed}}{M_{pl,N,y,Rd}} = \frac{900}{2814} = 0.320 < \alpha_{M,y} = 0.9$$

$$\frac{M_{z,Ed}}{M_{pl,N,z,Rd}} = \frac{700}{2814} = 0.249 < \alpha_{M,z} = 0.9$$

$$\frac{M_{y,Ed}}{M_{pl,N,y,Rd}} + \frac{M_{z,Ed}}{M_{pl,N,z,Rd}} = \frac{900}{2814} + \frac{700}{2814} = 0.569 < 1.0$$

Thus, the resistance for combined axial compression and uniaxial bending is adequate. The external design force and bending moment, and M-N interaction curve are plotted in Figures 17 - 19:

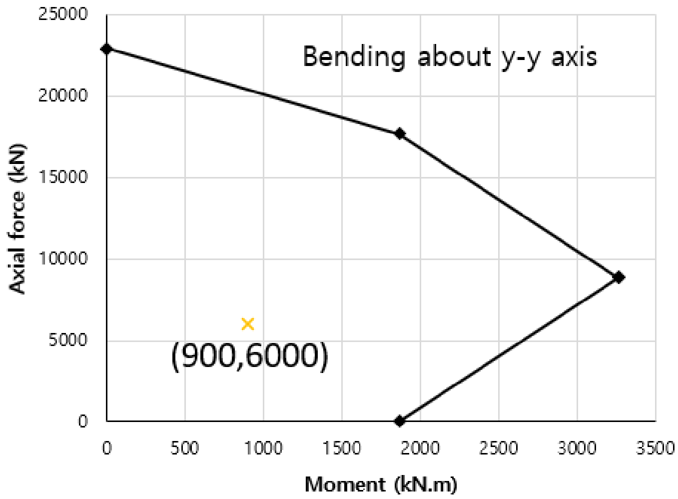


Figure 18 Design M-N interaction curve of PSRC about the y-y axis

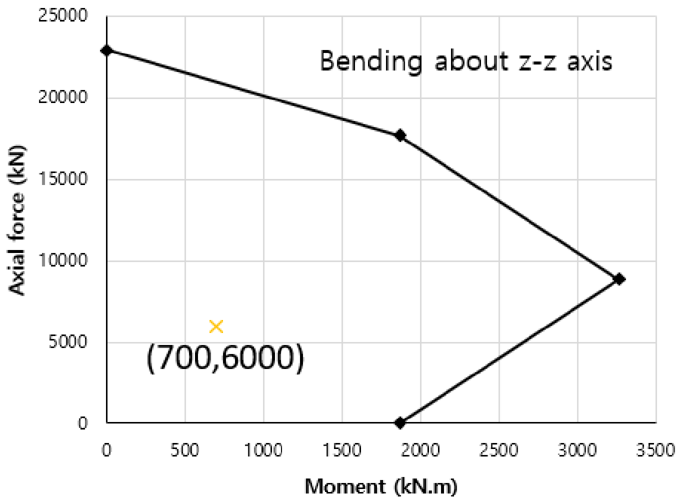


Figure 19 Design M-N interaction curve of PSRC about the z-z axis

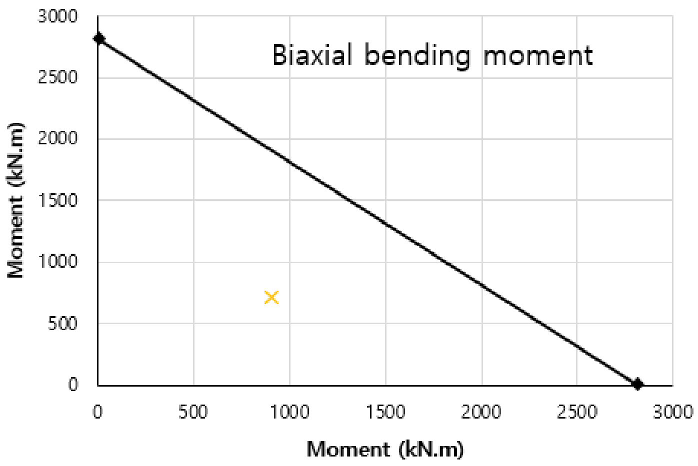


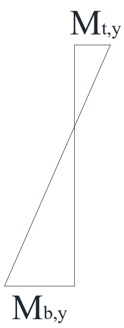
Figure 20 Design Biaxial Moment interaction curve of PSRC

5.3 4 x UKA 130x130x15 – S355 steel sections with UKA 70x70x6 S275 steel sections – 300mm c/c spg under construction loading

Properties with the subscript "ch" refer to that of the main chord: UKA 130x130x15 while the properties of the batten member UKA 70x70x6 are denoted with the subscript "b".

- Design Parameters:

Main angle sections	Grade S355, f_{yk}=355 N/mm^2
Batten angle sections	Grade S275, f_{yk}=275 N/mm^2
Column system length	L=8000 mm
Effective length	L_{eff} =8000 mm
Total design axial load	N_{Ed}=1000 kN



Design moment at top around y-y axis	$M_{t,y}$=80 kN.m
Design moment at bottom around y-y axis	$M_{b,y}$= 0 kN.m

- Material

  Angle sections Grade S355, f_y=355 N/mm^2

- Design strength and modulus of material

  $f_{yd} = f_y/\gamma_a = 355/1.0 = 355 \text{ N/mm}^2$

  $E_s = 210 \text{ GPa}$

- Sectional properties of the main chord angle: UKA 130x130x15

  Torsion constant of the gross cross-section of angle $I_{t,ch} = 27.56 \text{ cm}^4$

  Warping constant of the gross cross-section of angle $I_{w,ch} = 344.67 \text{ cm}^6$

  Elastic section modulus about u-u axis $W_{el,u,ch} = 100.3\ cm^3$

  Elastic section modulus about v-v axis $W_{el,v,ch} = 44.3\ cm^3$

  Gross area: A_{ch} = 3675 mm2

  Second moment of area about y-y axis $I_{y,ch} = 580.5 \text{ cm}^4$

  Second moment of area about z-z axis $I_{z,ch} = 580.5 \text{ cm}^4$

  Radius of gyration of the gross cross-section of angle about u-u axis $i_{u,ch} = 49.76 \text{ mm}$

  Radius of gyration of the gross cross-section of angle about v-v axis $i_{v,ch} = 24.79 \text{ mm}$

  Buckling length of the angle for torsional buckling $L_{T,ch} = L_b = 300 \text{ mm}$

  Shear center coordinates $u_{o,ch} = 53.75 \text{ mm}$

  Radius of gyration about the polar axis

  $$i_{0,ch} = \sqrt{i_y^2 + i_z^2 + u_{o,ch}^2 + v_{o,ch}^2 = 49.76^2 + 24.79^2 + 53.75^2 + 0} = \sqrt{59.81 \text{ cm}^2}$$

- Sectional properties of the batten angle: UKA 70x70x6

  Torsion constant of the gross cross-section of angle $I_{t,b} = 1.09\ \text{cm}^4$

  Elastic section modulus about u-u axis $W_{el,u,b} = 12.18\ cm^3$

  Elastic section modulus about v-v axis $W_{el,u,b} = 5.501\ cm^3$

  Gross area: A_b = 804 mm2

  Second moment of area about y-y axis $I_{y,b} = 37.8\ \text{cm}^4$

  Second moment of area about z-z axis $I_{z,b} = 37.8\ \text{cm}^4$

  Radius of gyration of the gross cross-section of angle about u-u axis $i_{u,b} = 27.38\text{mm}$

  Radius of gyration of the gross cross-section of angle about v-v axis $i_{v,b} = 13.81\ \text{mm}$

  Buckling length of the angle for torsional buckling $L_{T,b} = h_0 = 580\ \text{mm}$

- Elastic critical Euler buckling resistance of built-up member

  - Second moment of inertia of built-up section

    $$I_1 = 4\left(\frac{h_0}{2}\right)^2 A_{ch} + 4I_{ch}$$

    $$= 4\left(\frac{580.8}{2}\right)^2 (3675) \times 10^{-4} + 4(580.5) = 126290\ \text{cm}^4$$

  - Euler buckling load of built-up column

    $$i_0 = \sqrt{\frac{I_1}{4A_{ch}}} = \sqrt{\frac{126290 \times 10^4}{4 \times 3675}} = 293\ \text{mm}$$

    $$\lambda = \frac{L}{i_0} = \frac{8000}{293} = 27.3$$

    As λ = 27.3 < 75, efficiency factor $\mu = 1.0$

    $$I_{eff} = 4\left(\frac{h_0}{2}\right)^2 A_{ch} + 4\mu I_{ch}$$

    $$= 4\left(\frac{580.8}{2}\right)^2 (3675) \times 10^{-4} + 4(1.0)(580.5) = 126290\ \text{cm}^4$$

    $$N_{cr} = \frac{\pi^2 E I_{eff}}{L^2} = \frac{\pi^2 \times 210000 \times 126290 \times 10^4}{8000^2} \times 10^{-3} = 40899\ \text{kN}$$

- Characteristic plastic resistance of built-up member

  $N_{pl,Rd} = 4A_{ch}f_{yd} = 4(3675)(355) \times 10^{-3} = 5218.5 \text{ kN}$

  $M_{pl,Rd} = 2A_{ch}f_{yd}h_0 = 2(3675)(355)(580.8) \times 10^{-6} = 1513.4 \text{ kN·m}$

- Relative slenderness ratio

  $$\bar{\lambda} = \sqrt{\frac{N_{pl,Rd}}{N_{cr}}} = \sqrt{\frac{5218.5}{40898}} = 0.35$$

- Buckling curves and buckling reduction factors

  For a bolted battened truss, the buckling curves about both axes are “c”. Thus, the imperfection factor is $\alpha = 0.49$.

  $$\Phi = 0.5\left[1 + \alpha(\bar{\lambda} - 0.2) + \bar{\lambda}^2\right] = 0.5[1 + 0.49 \times (0.350 - 0.2) + 0.350^2] = 0.598$$

  $$\chi = \min\left(\frac{1}{\Phi + \sqrt{\Phi^2 - \bar{\lambda}^2}}, 1.0\right) = 0.923$$

  $N_{b,Rd} = \chi f_{yd}(4A_{ch}) = 4817 \text{ kN}$

- Buckling interaction factor

  $$\psi_y = \frac{0}{80} = 0$$

  $C_{my} = \min(0.6 + 0.4\psi, 0.4) = 0.4$

  As $\bar{\lambda} = 0.35 < 1.0$ and the built-up member is not susceptible to lateral torsional bucklin g, the interaction factor k_{yy} is given by:

  $$k_{yy} = C_{my}\left(1 + 0.6\bar{\lambda}\frac{N_{Ed}}{N_{b,Rd}}\right) = 0.4\left(1 + 0.6 \times 0.35\frac{1000}{4817}\right) = 0.417$$

- Flexural buckling of built-up member under compression and uniaxial bending

  $$\frac{N_{Ed}}{N_{b,Rd}} + k_{yy}\frac{M_{Ed}}{M_{pl,Rd}} = \frac{1000}{4817} + 0.417\frac{80}{1513} = 0.23 < 1.0$$

- Design forces on chords and battens

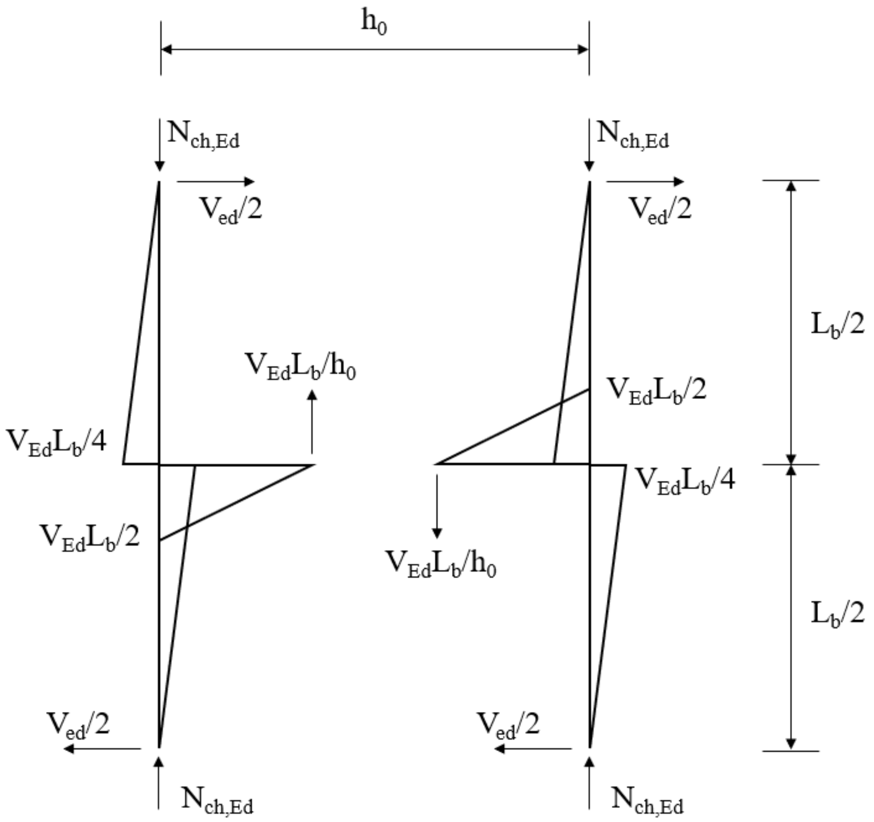


Figure 21 Bending moment diagram of the end span of a single batten panel

- First order bending moment around y-y axis at mid height

$$M_{Ed,y,c} = \frac{(80 + 0.0)}{2} = 40 \text{ kNm}$$

- Shear stiffness of the built-up column

There is one layer of battens for each panel of main angle chords: $n = 1.0$

$$S_v = \min\left(\frac{24E_s(2I_{y,ch})}{L_b^2\left[1 + \frac{2(2I_{y,ch})}{n(2I_{y,b})}\frac{h_0}{L_b}\right]}, \frac{2\pi^2 E_s(2I_{y,ch})}{L_b^2}\right)$$

$$= \min\left(\frac{24 \times 210 \times (2 \times 580.5)}{30^2 \times \left[1 + \frac{2\times(2\times580.5)}{2\times37.8} \times \frac{580.8}{300}\right]} \times 10^2, \frac{2 \times \pi^2 \times 210 \times 1136}{30^2} \times 10^2\right)$$

$$= \min(10753 \text{ kN}, 534735 \text{ kN}) = 10753 \text{ kN}$$

- Moment at column mid height

Bow imperfection is given in cl. EN1993-1-1 6.4.1(1) by $e_0 = L/500$

$$e_0 = \frac{8000}{500} = 16\text{mm}$$

$$M_{L/2,y,Ed} = \frac{N_{Ed,c}e_0 + M_{Ed,y,c}}{1 - \frac{N_{Ed}}{N_{cr}} - \frac{N_{Ed}}{S_v}} = \frac{1000 \times 16 \times 10^{-3} + 40}{1 - \frac{1000}{40899} - \frac{1000}{10753}} = 63.5 \text{ kN} \cdot \text{m}$$

- Design cross-sectional bending moment

$$M_{Ed} = \max(|M_{b,y}|, |M_{L/2,y}|, |M_{t,y}|) = \max(0, 63.5, 80) = 80 \text{ kN·m}$$

The cross section at the top governs the design

- Design cross-sectional shear force

The shear force at any section along the built up column is given by

$$V_{Ed} = \frac{dM_{Ed,y,c}}{dx} + N_{Ed,c}\frac{dw(x)}{dx}$$

where

$$\frac{dw(x)}{dx} = \frac{1}{\left(1 - \frac{N_{Ed}}{N_{cr}} - \frac{N_{Ed}}{S_v}\right)}\left(\frac{\pi}{L}e_0 \cos\left(\pi\frac{x}{L}\right) + \frac{dw_1}{dx}\right)$$

and

$\frac{dw(x)}{x}$	is	the slope of the column deflection at a length x away from the top of the column
$\frac{dw_1}{dx}$	is	the slope deflection of the column under first order forces at a length x away from the top of the column

For the given scenario, the first order slope of deflection at the top of the column is given by

$$\frac{dw_1}{dx} = \frac{M_{y,t}L}{6EI} = \frac{80 \times 8}{6 \times 210 \times 126290 \times 10^{-2}} = 4.02 \times 10^{-4}$$

The slope of the column deflection at the top of the column is given by

$$\frac{dw(x)}{dx} = \frac{1}{\left(1 - \frac{N_{Ed}}{N_{cr}} - \frac{N_{Ed}}{S_v}\right)}\left(\frac{\pi}{L}e_0 + \frac{dw_1}{dx}\right)$$

$$= \frac{1}{\left(1 - \frac{1000}{40899} - \frac{1000}{10753}\right)}\left(\pi\frac{16}{8000} + 4.02 \times 10^{-4}\right) = 7.575 \times 10^{-3}$$

Hence, the shear force at the top of the column is given by

$$V_{Ed} = \frac{M_{t,y} - M_{b,y}}{L} + N_{Ed,c}\frac{dw(x)}{dx}$$

$$= \frac{80}{8} + 1000 \times (7.575 \times 10^{-3}) = 17.58 \text{ kN}$$

- Design axial force in a single main vertical chord

$$N_{ch,Ed} = 0.25N_{Ed} + 0.5\frac{M_{Ed}h_0A_{ch}}{2I_{eff}}$$

$$= 0.25 \times 1000 + 0.5\frac{80 \times 580 \times 73.50 \times 10^{-7}}{2 \times 125899 \times 10^{-8}} = 317.7 \text{ kN}$$

- Bending moment in main vertical chord at top panel

$$M_{ch,Ed} = 0.5 \times \frac{V_{Ed}L_b}{4} = 0.5 \times \frac{17.58 \times 300 \times 10^{-3}}{4} = 0.659 \text{ kN} \cdot \text{m}$$

- Shear force in main vertical chord at top panel

$$V_{ch,Ed} = 0.5\frac{V_{Ed}}{2} = 0.5\frac{17.58}{2} = 4.394 \text{ kN}$$

- Bending moment in horizontal batten angle at top panel

$$M_{b,Ed} = 0.5 \times \frac{V_{Ed}L_b}{2} = 0.5 \times \frac{17.58 \times 300 \times 10^{-3}}{2} = 1.318 \text{ kN} \cdot \text{m}$$

- Shear force in horizontal batten angle at top panel

$$V_{b,Ed} = 0.5 \times \frac{V_{Ed}L_b}{h_0} = 0.5 \times \frac{17.58 \times 300 \times 10^{-3}}{580.8 \times 10^{-3}} = 4.54 \text{ kN}$$

- Main vertical chord capacity check

  - Section classification

$c = (130 - 15) = 115$ mm

$$c/t = \frac{115}{15} = 7.7 < 10\varepsilon = 8.1$$

Section is class 2 under compression

  - Elastic critical force for torsional-flexural buckling

$I_{v,ch} = 225.9\ \text{cm}^4$

$$N_{cr,v,ch} = \frac{\pi^2 E I_{v,ch}}{L_b^2} = \frac{\pi^2 \times 210 \times 225.9}{30^2} \times 10^2 = 52024\ \text{kN}$$

$$N_{cr,T,ch} = \frac{1}{i_o^2}\left(GI_t + \frac{\pi^2 E I_w}{l_T^2}\right)$$

$$= \frac{1}{59.81} \times \left(79.3 \times 27.56 + \frac{\pi^2 \times 210 \times 344.67}{30^2}\right) \times 10^2 = 4982\ \text{kN}$$

$$\beta = 1 - \left(\frac{u_{0,ch}}{i_{o,ch}}\right)^2 = 1 - \frac{53.75^2 \times 10^{-2}}{59.81} = 0.517$$

$$N_{cr,TF,ch} = \frac{N_{cr,a}}{2\beta}\left[1 + \frac{N_{cr,T}}{N_{cr,a}} - \sqrt{\left(1 - \frac{N_{cr,T}}{N_{cr,a}}\right)^2 + 4\left(\frac{y_o}{i_o}\right)^2 \frac{N_{cr,T}}{N_{cr,a}}}\right]$$

$$= \frac{52024}{2 \times 0.517}\left[1 + \frac{4982}{52024} - \sqrt{\left(1 - \frac{4982}{52024}\right)^2 + 4 \times \frac{53.75^2 \times 10^{-2}}{59.81} \times \frac{4982}{52024}}\right]$$

$$= 4750\ \text{kN}$$

$$N_{cr,u,ch} = \frac{\pi^2 E I_{u,ch}}{L_b^2} = \frac{\pi^2 \times 210 \times (36.75 \times 4.98^2)}{30^2} \times 10^2 = 209553\ \text{kN}$$

  - Non-dimensional slenderness

$$\bar{\lambda}_T = \sqrt{\frac{A_{cr} f_y}{\min[N_{cr,T}, N_{cr,TF}]}} = \sqrt{\frac{36.75 \times 355 \times 10^{-1}}{\min[4982,4750]}} = 0.524$$

$$\Phi = 0.5\left[1 + \alpha\left(\bar{\lambda}_T - 0.2\right) + \bar{\lambda}_T{}^2\right]$$

$$= 0.5 \times [1 + 0.34 \times (0.524 - 0.2) + 0.524^2] = 0.69$$

$$\chi = \left(\Phi + \sqrt{\Phi^2 - \bar{\lambda}_T{}^2}\right)^{-1} = \left(0.69 + \sqrt{0.69^2 - 0.515^2}\right)^{-1} = 0.87$$

$$N_{b,Rd} = \chi A_{cr} f_{yd} = 0.87 \times 36.75 \times 355 \times 10^{-1} = 1139 \text{ kN}$$

- Flexural Torsional Buckling Resistance utilisation

$$\frac{N_{ch,Ed}}{N_{b,Rd}} = \frac{317.7}{1145} = 0.28 \leq 1.0$$

- Shear utilisation

$$\frac{V_{Ed}}{V_{pl,Rd}} = \frac{4.394 \times 10^3}{355/\sqrt{3} \times (130 \times 15)} = 0.0110 < 0.5$$

Section is not under high shear

- Bending moment check

Bending moments about angle sections should be decomposed into the u-u and v-v components

$$M_{ch,u,Ed} = M_{ch,Ed} \sin 45° = 0.466 \text{ kN} \cdot \text{m}$$

$$M_{ch,v,Ed} = M_{ch,Ed} \cos 45° = 0.466 \text{ kN} \cdot \text{m}$$

- Cross section compression and bending moment utilisation

$$\frac{N_{Ed}}{f_y A} + \frac{M_{a,u,Ed}}{f_y W_u} + \frac{M_{a,v,Ed}}{f_y W_v} = \frac{317.7}{355 \times 3675} + \frac{0.466 \times 10^6}{355 \times 100.3 \times 10^3} + \frac{0.466 \times 10^6}{355 \times 44.3 \times 10^3}$$

$$= 0.244 + 0.013 + 0.031 = 0.286 < 1.0 \text{ [OK]}$$

- Non-dimensional slenderness ratio:

The non-dimensional slenderness ratio of an equal-angle is calculated in accordance with cl.BS5950-1-2000: B.2.9.2 due to lack of guidance in Eurocode 3.

$$\lambda_v = \frac{L_b}{i_v} = \frac{300}{24.79} = 12.1$$

For equal angles, $\psi_a = 1$

$$v_a = \left(\sqrt{1 + \left(\frac{4.5\psi_a^2}{\lambda_v}\right)} + \left(\frac{4.5\psi_a^2}{\lambda_v}\right)\right)^{-0.5} = 0.805$$

$$g = \sqrt{1 - \frac{I_v}{I_u}} = 0.86$$

$$\phi_a = \frac{W_{el,u,ch} g}{\sqrt{A_{ch} I_{T,ch}}} = 2.71$$

$$\bar{\lambda}_{LT} = 0.72 v_a \sqrt{\frac{f_y}{E}\phi_a \lambda_v} = 0.137$$

As $\bar{\lambda}_{LT} = 0.137 < 0.4$, the main vertical chord is not susceptible to lateral torsional buckling.

$$\overline{\lambda_u} = \sqrt{\frac{N_{pl,Rd}}{N_{cr,u}}} = \sqrt{\frac{355 \times 3675}{209890 \times 10^3}} = 0.08$$

As $N_{cr,TF} < N_{cr,T} < N_{cr,v}$, buckling about the minor axis is governed by torsional-flexural buckling.

$$\overline{\lambda_{TF}} = \sqrt{\frac{N_{pl,Rd}}{N_{cr,TF}}} = \sqrt{\frac{355 \times 3675}{4925 \times 10^3}} = 0.514$$

- Buckling interaction factors

$$\psi_u = \psi_v = 0$$

$$C_{mu} = C_{mv} = 0.4$$

$$k_{uu} = C_{my}\left(1 + 0.6\bar{\lambda}_u \frac{N_{Ed}}{N_{b,u,Rd}}\right) = 0.4\left(1 + 0.6 \times 0.08 \frac{317.7}{355 \times 3675}\right) = 0.4$$

$$k_{uv} = 0.8 k_{uu} = 0.32$$

$$k_{vv} = C_{mv}\left(1 + 0.6\bar{\lambda}_{TF} \frac{N_{Ed}}{N_{b,FT,Rd}}\right) = 0.4\left(1 + 0.6 \times 0.514 \frac{317.7}{1145}\right) = 0.434$$

- Flexural torsional buckling about the minor axis under compression and biaxial bending moments

$$\frac{N_{Ed}}{N_{FT,Rd}} + k_{uv}\frac{M_{u,Ed}}{M_{u,b,Rd}} + k_{vv}\frac{M_{u,Ed}}{M_{u,Rd}}$$

$$= \frac{317.7}{1145} + 0.32\ \frac{0.466 \times 10^6}{355 \times 100.3 \times 10^3} + 0.434\ \frac{0.466 \times 10^6}{355 \times 44.3 \times 10^3} = 0.296 < 1.0$$

○ Horizontal batten angle

- Section classification

$c = (70 - 6)$ = 64 mm

$$c/t = \frac{64}{6} = 10.7 < 14\varepsilon = 12.9$$

Section is class 3 under compression

- Shear utilisation

$$\frac{V_{b,Ed}}{V_{b,pl,Rd}} = \frac{4.54 \times 10^3}{275/\sqrt{3} \times (70 \times 6)} = 0.068 < 0.5$$

Section is not under high shear

- Cross-section bending moment utilisation

Bending moments about angle sections should be decomposed into the u-u and v-v components

$$M_{b,u,Ed} = M_{b,Ed} \sin 45° = 0.932\ \mathrm{kN \cdot m}$$

$$M_{b,v,Ed} = M_{b,Ed} \cos 45° = 0.932\ \mathrm{kN \cdot m}$$

$$\frac{M_{a,u,Ed}}{f_y W_u} + \frac{M_{a,v,Ed}}{f_y W_v} = \frac{0.932 \times 10^6}{275 \times 12.18 \times 10^3} + \frac{0.932 \times 10^6}{275 \times 5.501 \times 10^3}$$

$$= 0.278 + 0.671 = 0.95 < 1.0 \text{ [OK]}$$

- Non-dimensional slenderness for lateral torsional buckling

$$\lambda_v = \frac{L}{i_v} = \frac{580.8}{13.81} = 42.06$$

for equal angles, $\psi_a = 1$

$$v_a = \left(\sqrt{1 + \left(\frac{4.5\psi_a^2}{\lambda_v}\right)} + \left(\frac{4.5\psi_a^2}{\lambda_v}\right)\right)^{-0.5} = 0.929$$

$$g = \sqrt{1 - \frac{I_v}{I_u}} = 0.86$$

$$\phi_a = \frac{W_{el,u}g}{\sqrt{AI_T}} = 3.553$$

$$\bar{\lambda}_{LT} = 0.72v_a\sqrt{\frac{f_y}{E}\phi_a\lambda_v} = 0.306$$

As $\bar{\lambda}_{LT} = 0.306 < 0.4$, the member is not susceptible to lateral torsional buckling.

- Bending moment utilisation of effective area at connection region

Suppose the batten angles are connected using a single row of two M16 bolts, shown in Figure 22, the effective area of the angle is calculated in accordance with cl. EN1993-1-8:2005 3.10.3(2), as shown in Figure 23.

$$d_0 = 18 \text{ mm} \Rightarrow \frac{p_1}{d_0} = \frac{60}{18} = 3.33$$

For $2.5 < p_1/d_0 < 5.0$,

$$\beta_2 = \frac{(p_1/d_0) - 2.5}{5.0 - 2.5}0.3 + 0.4 = \frac{3.33 - 2.5}{5.0 - 2.5}0.3 + 0.4 = 0.50$$

$$A_{net} = A_g - d_0 t = 804 - 18 \times 6 = 696 \text{ mm}^2$$

$$a_{2,eff} = \frac{\beta A_{net}}{t} - (a_1 - d_0 - t) = \frac{0.5(696)}{6} - (70 - 18 - 6) = 12 \text{ mm}$$

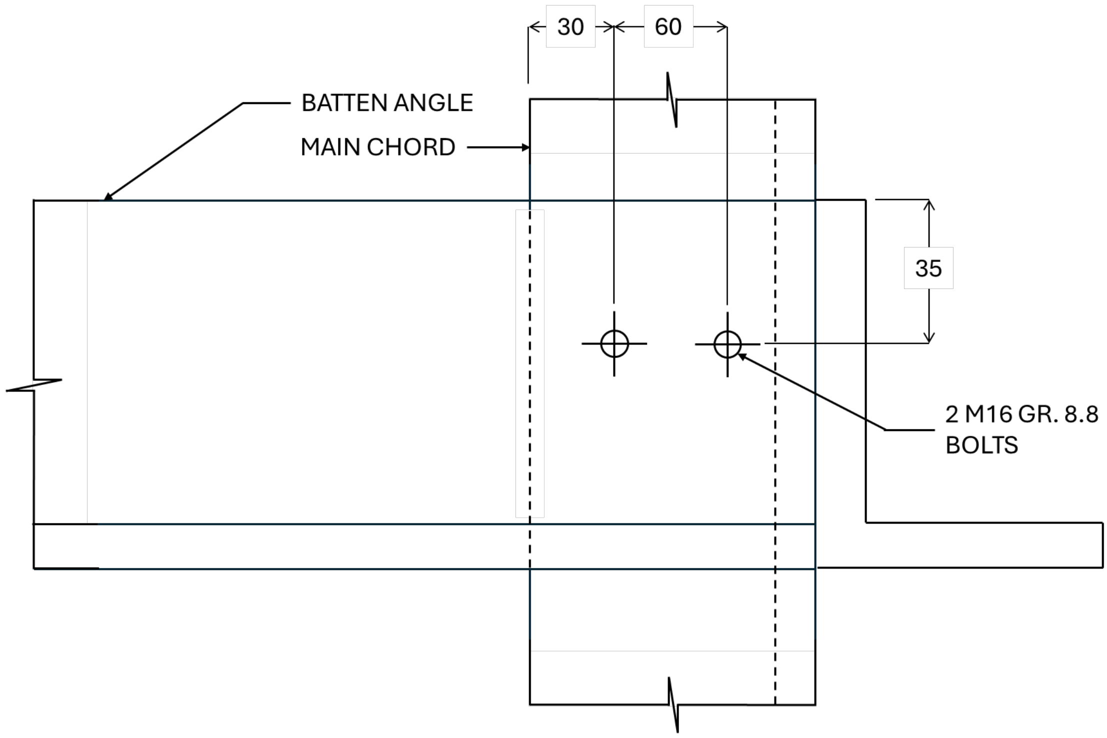


Figure 22 Horizontal batten – vertical chord bolted connection in Example 5

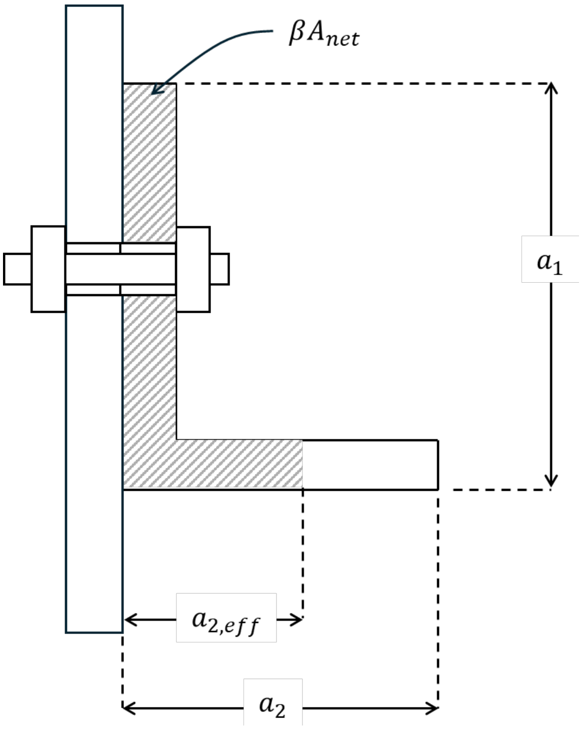


Figure 23 Effective area of batten angle at connection in Example 5

The elastic section modulus of the effective area about the y-y axis is given by:

$W_{el,eff} = 5.477 \text{ cm}^3$

$M_{el,Rd} = 275 \times 5.477 \times 10^{-3}$=1.51 kN·m

$$\frac{M_{Ed}}{M_{el,Rd}} = \frac{1.318}{1.51} = 0.873$$

As the utilisation ratio $M_{Ed}/M_{el,,Rd} = 0.873 < 1.0$, the horizontal batten is adequate.